Filter Comparison in Wavelet Transform of Still Images

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***Abstract* – This paper discusses the features of wavelet filters in compression of still images and characteristic that they show for various image content and size. The aim of this work is to create a palette of functions (filters) for implementation of wavelet in still image processing and to emphasize the advantage of this transformation relating to today's methods. Filters taken in the test are some of the most used: Haar filter (as the basis), orthogonal Daubechies' and Battle-Lemarie filters such are Least Asymmetric, Shannon and Coiflet, and symmetric Biorthogonal filter. All these filters give various performances for images of different content. Objective and subjective picture quality characteristics of images coded using wavelet transform with different filters are given. The comparison between JPEG coded picture and the same picture coded with wavelet transform is given. For higher compression ratios it is shown that wavelet transform has better *S/N* than JPEG.**

I. INTRODUCTION

The wavelet transform is operation that decomposes a signal into components at different scales, [1, 10]. Wavelet transform of still images is based on multiresolution analysis and splitting the image signal in low and high frequency components through convolution of the signal with dilated filter. The distinction between discrete cosine transform (DCT) used for JPEG and wavelet transform is in shape of function that is used for transformation. In wavelet transform time limited functions are used. For elementary wavelet transform we use scaling and wavelet function. Scaling as a low pass filter, and wavelet as a high pass filter. In comparison with Fourier transform which represent a signal as a superposition of sinusoids on different frequencies, where the coefficients measure the contribution of the sinusoids at these frequencies, wavelet transform represents a sum of wavelets on different locations and scales, where the wavelet coefficients quantify the strength of the contribution of the wavelets at these location and scales, [2].

Wavelet transform has a wide range of statistical properties. They can have high differentiability or can be chaotic with properties much like distributions. Some application can thus determine the characteristics of the basis function that would be the most suitable for characteristic data (image). In most cases smoother wavelets can give very similar results for images with various transitions of dark and bright, but the results can vary more considerably with higher compression rate and higher resolution, [3].

Although, deriving wavelet functions requires wide and complex mathematical skills, there are many new and older wavelets with different characteristic, [4]. Here are described some of the most used. This chapter shows fundamental features of the functions in time and frequency domain.

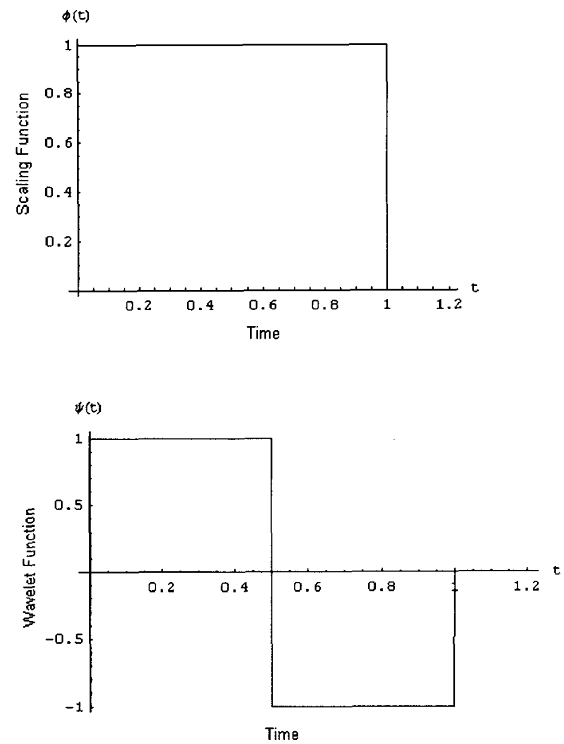


Fig. 1. Scaling and wavelet Haar functions

The first mentioned is Haar function (filter) with sharp edges. Fig. 1 shows scaling and wavelet Haar function. The main problem of this function is lack of smoothness, because the function requires a large number of terms in expansion. However, decomposition in the Haar basis, as in any wavelet basis, eliminates high frequency terms when the input sequence is constant. That's way Haar function is often used for images with high contrast of black and white.

Haar filter represents special case of Daubechies filter family. Haar filter is actually Daubechies filter of order 1. This family is often called *compactly supported orthonormal filters*. These filters have no explicit expressions except for Haar. The construction is based on solving the frequency response function for the filter coefficients satisfying orthogonality and moment conditions. Daubechies approach to finding scaling and wavelet function is to first determine the finite number of coefficients such that orthogonality and smoothness or moment conditions are guaranteed. The main feature of Daubechies family is orthogonality and asymmetry. Fig. 2 shows scaling and wavelet Daubechies function of order 4.

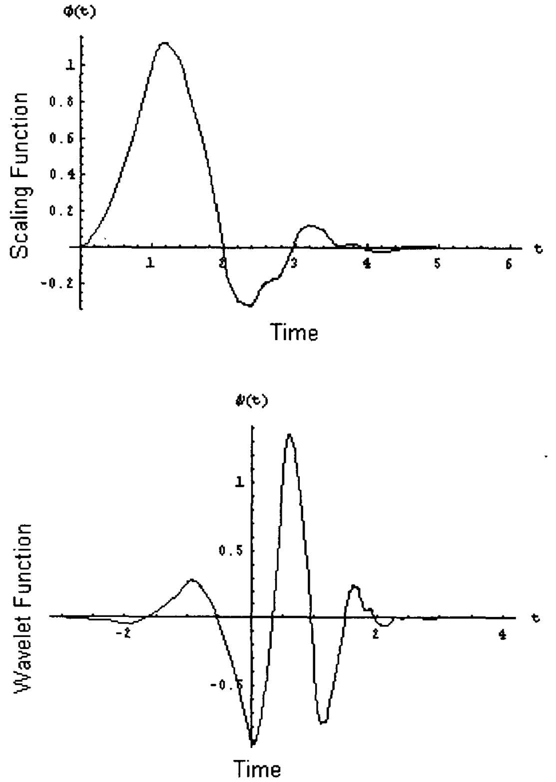


Fig. 2. Scaling and Wavelet Daubechies functions

Battle-Lemarie filters are very important for use in wavelet transform. Some of them are orthogonal and provide improved computational performance. Spline filter is one of the family and Fig. 3 and Fig. 4 show how the slope of the filter in frequency domain depends on order of the filter. It is shown that increasing the order of the filter the sharper slopes of the frequency response are provided. Coiflet filters show more symmetry than Daubechies but they are not orthogonal. The Least Asymmetric filter shows also non-orthogonal features and is closest to symmetry among Battle-Lemarie filters. Biorthogonal filters are based on the biorthogonality condition. Orthogonality of filter gives better computational performance, and symmetry is often desirable because of the linear phase of the transfer function.

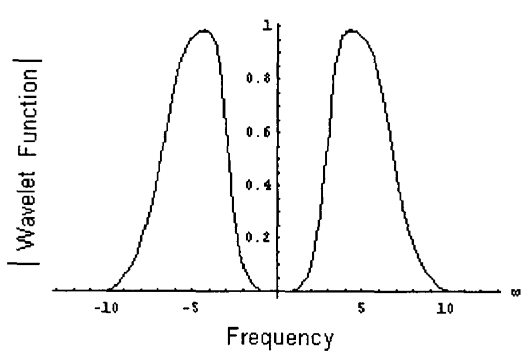


Fig. 3. Frequency response of Spline scaling function of order 2

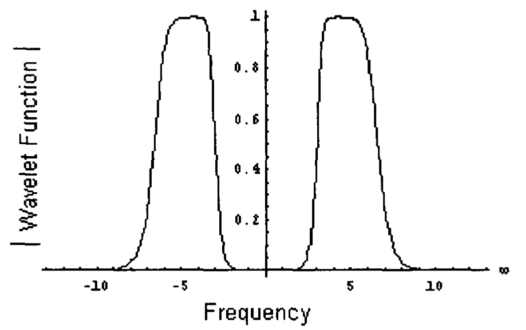


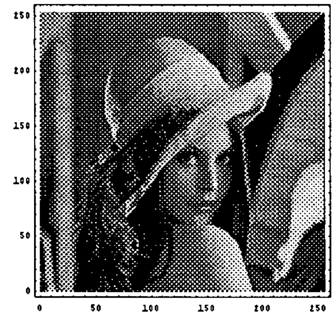
Fig. 4. Frequency response of Spline wavelet function of order 5

II. COMPARISON OF WAVELET FILTERS

Selection of filter depends of few basic parameters such as: type of function, order of decomposition, order of filter, kind of multiresolution analysis, content of an image, [6]. These parameters are depended on each other. For example, order of decomposition depends on order of filter and reversal, the order of filter depends on order of decomposition and type of function. In most cases increasing the order of function increases the space which function takes in time domain. With order of decomposition we determine the lowest resolution of an image in wavelet domain; so if the time window of function pass beyond the time needed for analysis of the lowest decomposition level., wavelet transform makes no sense any more. The quality of the image can only be degraded. In further text will be shown that higher number of decompositions gives better performance of compress-ion, especially for large compression ratio. However, increasing the number of filter gives us usually a smoother function and sharper slopes of the filter in frequency domain. So, choosing the filter order and the order of decomposition must be compromise. Evaluation of the best order of the filter for an image needs large number of tests and out of the results (sample for image of Lena is shown in Table 1) of the test here is given the comparison of the filters with parameters that give the best results for each filter family.

TABLE I
COMPARISON OF S/N RATIO FOR DAUBECHIES FILTER USING DIFFERENT ORDER OF FILTER

Order of filter	Number of decomposition	S/N ratio [dB]
1	8	31.2419
2	7	32.6879
3	6	33.1644
4	5	33.2533
5	5	33.0011
6	5	32.9561
7	5	32.9187
8	5	32.8839
9	4	32.8087



Original Image

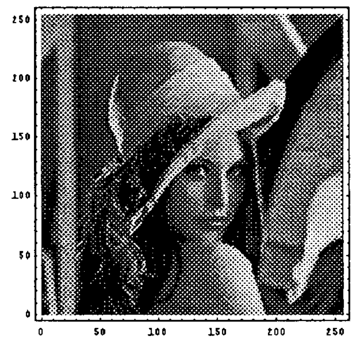
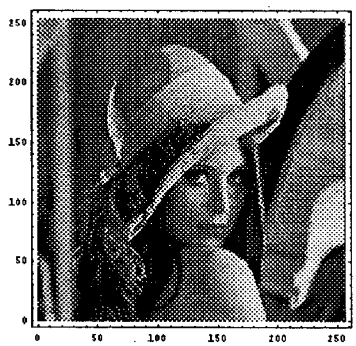
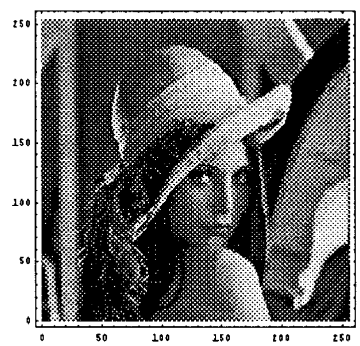
Haar filter, 5 decomp. S/N 31.2427			Daubechies filter, order 4, 5 decomp. S/N 33.2533
Least Asymm. filter, order 5, 5 decomp. S/N 33.4504			Coiflet filter, order 6, 4 decomp. S/N 33.5476
Biorthogonal filter, order 2, 6 decomp S/N 33.3108			Shannon filter, order 25, 3 decomp. S/N 23.2320

Fig. 5. Comparison of different filters for Lena image (resolution 256×256 pixels; compression ratio 1:10)

The *S/N* ratio is defined as:

$$\frac{S}{N} = 20\log\frac{2^n-1}{RMS}\ \text{[dB]},\ \text{where}\ RMS=\sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i-\overline{x_i})^2} \qquad (1)$$

x_i are pixels of original image and $\overline{x_i}$ are pixels of compressed image. First example is the Lena image with resolution of 256×256 pixels. The original image is shown on Fig. 5.

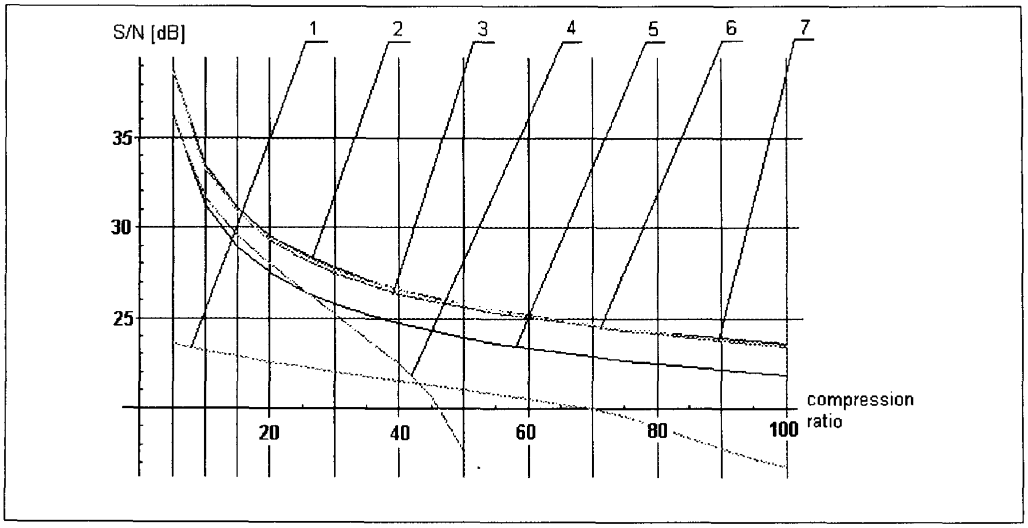


Fig. 6. *S/N* graph for the Lena image (resolution 256×256 pixels) 1. Shannon filter, 2. Least Asymmetric filter, 3. Biorthogonal filter, 4. JPEG, 5. Haar filter, 6. Daubechies filter, 7. Coiflet filter

Other images on Fig. 5 are compressed with compression ratio 1:10 for each of the filters. Here is used: Haar, Daubechies, Least Asymmetric, Coiflet, Biorthogonal and Shannon filter The best result is given by Least Asymmetric filter of order 5, and also very close performance give Daubechies and Coiflet filters. Shannon filter has very flat characteristic by increasing the compression ratio. For images with no requirements for higher quality Shannon filter could be used with very high compression level. With increasing the order of that filter quality my be better, but with higher order of filter, the number of decomposition falls, and the curve in *S/N* graph is under the curve of Haar filter, Fig. 6. Increasing the compression ratio all filters show better *S/N* than JPEG. Smooth filters (Daubechies, Least Asymmetric, Coiflet) together with Biorthogonal give similar results. Similar result is given also by Spline and Meyer filters. For compression ratio of 1/10 Spline filter of order 2 gives *S/N* 33.3129 dB and Meyer of order 2 gives 32.8125 dB. The distinction between various decomposition numbers is shown with Haar filter. On the Fig. 5 Haar filter of 5 decomposition shows somewhat lower S/N than the Haar with 8 decompositions shown in Table 1.

Next example is Zebra image with resolution of 256×256 pixels. Fig. 8 shows the comparison of six filters used also for the Lena image with compression ratio of 1:10. This picture contains sharp edges of white and black, and huge black area, there is no a lot of smooth transitions, and details are suppressed. The result for the Zebra image differs from that for Lena image by 5 dB in quality, Fig. 7. The quality of the compressed image depends of filter used for transformation and of the content of an image. This image contains a lot of flat (smooth) area with sharp edges of white and black. So, all smoother shape functions are closer to Haar function.

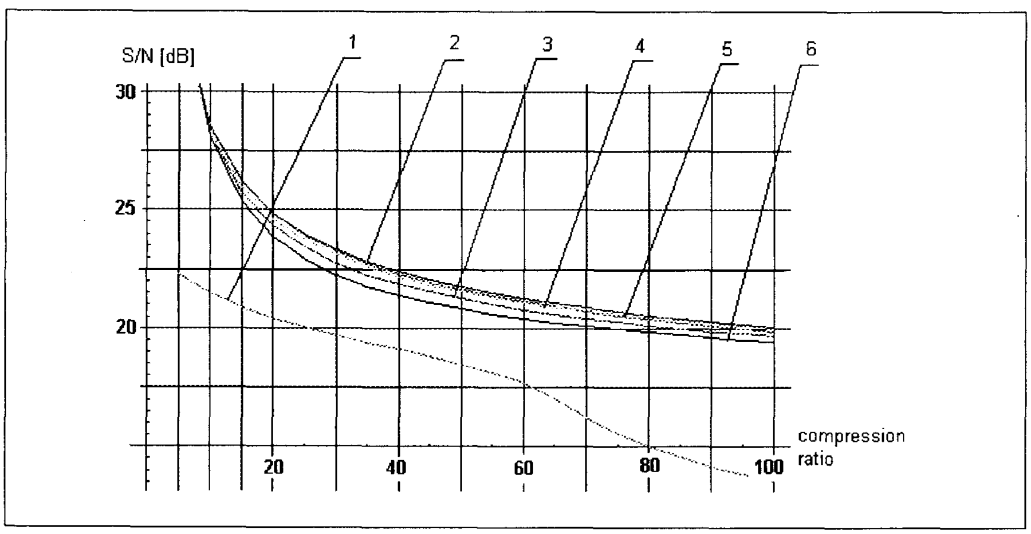
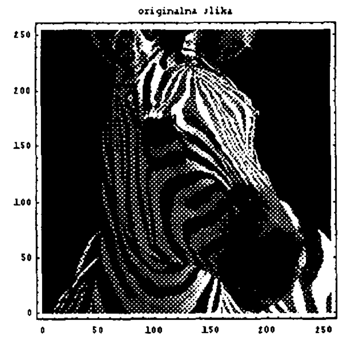


Figure 7: *S/N* graph for the Zebra image (resolution 256×256 pixels) : 1.Shannon filter, 2. Least Asymmetric filter, 3. B i orthogonal filter, 4. Daubechies filter, 5. Cioflet filter, 6. Haar filter



Original Image

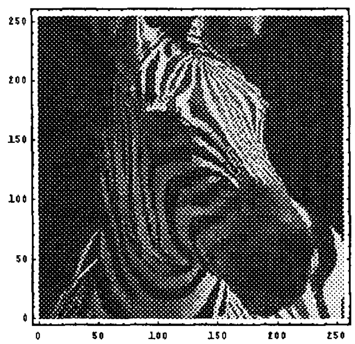
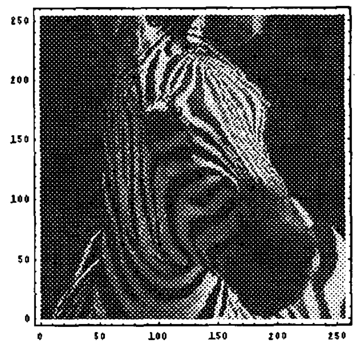
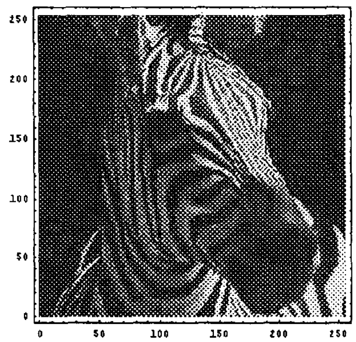
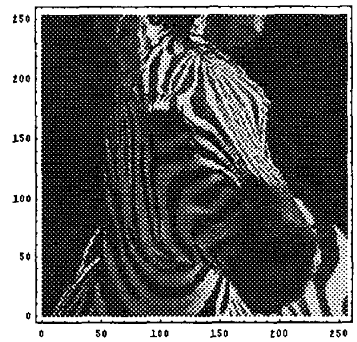
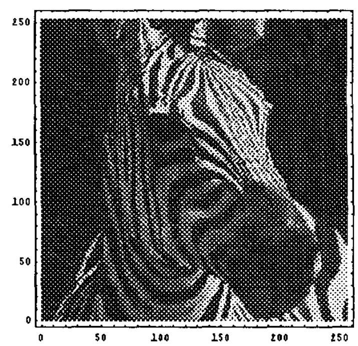
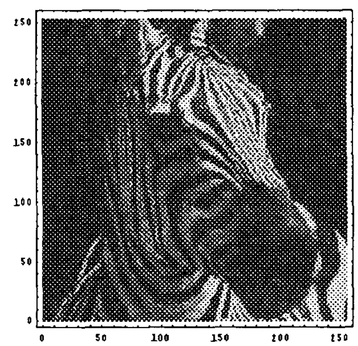
Haar filter, 5 decomp. S/N 28.1047			Daubechies filter, order 4, 5 decomp. S/N 28.1977
Least Asymm. filter, order 5, 5 decomp. S/N 28.4831			Coiflet filter, order 6, 4 decomp. S/N 28.5292
Biorthogonal filter, order 2, 6 decomp S/N 28.2055			Shannon filter, order 25, 3 decomp. S/N 21.5234

Fig. 8. Comparison of different filters for the Zebra image (resolution 256x256 pixels; compression ratio 1:10)

Transformation with Spline filter for Zebra image of order 2 has quality of 28.4181 dB for compression ratio of 1:10 and Mayer filter of order 2 gives S/N of 28.1035, what is similar to smooth function such as Coiflet, Daubechies and Least Asymmetric. Fig. 9 shows Lena and Zebra images for the compression ratio 1:100. The wavelet compression in both cases show better results than JPEG method. The resolution of 256×256 pixels can show that wavelet transform has some weaker or same performance for low compression ratios than DCT (JPEG), but testing the images of higher resolutions shows much better performance of wavelet transform.

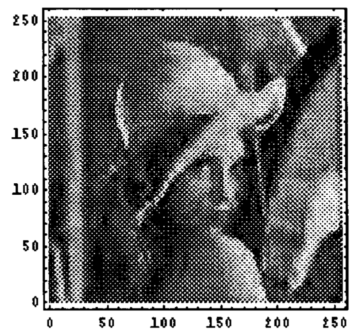
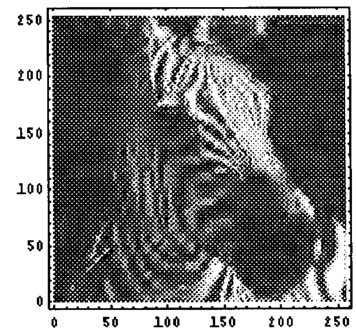
Least Asymm. filter, order 5, 5 decomp. S/N 23.5783			Least Asymm. filter, order 5, 5 decomp. S/N 20.0693

Fig. 9. Lena and Zebra image with compression ratio of 1:100

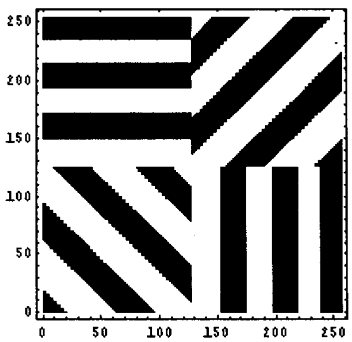
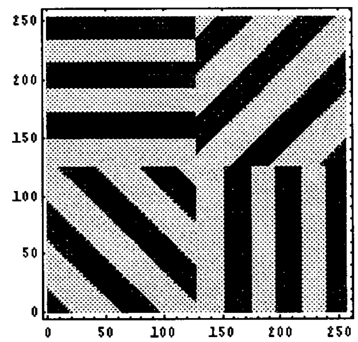
Original image 256×256 pixels			Least Asymm. filter, order 5, 5 decomp. S/N 27.5438 dB

Fig. 10. Comparison of Haar and Least Asymmetric filter used for transformation of artificial image; compression ratio 1:10

For the example, Biorthogonal filter of order 2 and wavelet transform for the image with resolution of 512×512 pixels shows *S/N* ratio 2 dB better than the transform of image with resolution of 256×256 pixels. Using DCT we do not expect some improvement on higher resolutions because it transforms the block of 8×8 pixels independent of the image size. So, wavelet performance over JPEG is much better on higher resolutions and with images that contain more details.

How the quality of compressed image depends of a content of an image and on used filter shows Fig. 10. The image used for the test is artificial image, 8-bit grayscale, but only with pure white and black components. In this case the best result is given by Haar filter which is for the Lena and Zebra images approximately 2 dB lower comparing with Least Asymmetric filter in *S/N* graph, Fig. 6 and Fig. 7. This performance of Haar filter comes from its shape and for compression ratio of 1:10 shows image identical to original image. The rectangular shape of filter coincides with rectangular transitions on image. This way the performance of Haar filter stands better up to compression ratio of 1:50. On higher compression ratio the quality of image is close for both filters.

III. CONCLUSION

Although, wavelet transform of still images is in phase of development, the results given by various researches give much better performance of this method than any other conventional method for the image compression. The main advantage of wavelet transform is possibility of compression with high compression ratios. The results given here show that wavelet transform has satisfying image quality, especially for images of higher resolution. The comparison of various filters was used to determine possible implementation of these filters in compression of still images. Wavelets can be main substitute for today's still image processing methods, like JPEG method is in the present time. In the framework of new JPEG2000 standard the wavelet transform is very important candidate for the still image compression.

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