Image Compression Using Wavelets

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Abstract **- Discrete wavelet transform (DWT) represents image as a sum of wavelet functions (wavelets) on different resolution levels. Basis for wavelet transform can be composed of any function that satisfies requirements of multiresolution analysis. It means that there exists a large selection of wavelet families depending on the choice of wavelet function. The choice of wavelet family depends on the application. In image compression application this choice depends on image content. This paper will provide fundamentals of wavelet based image compression. The options for wavelet image representations are tested. The results of image quality measurements for different wavelet functions, image contents, compression ratios and resolutions are given.**

I. INTRODUCTION

Discrete Wavelet Transform (DWT) can be efficiently used in image coding applications because of their data reduction capabilities. Unlike the case of Discrete Cosine Transform (DCT) which basis is composed of cosine functions, basis of DWT can be composed of any function (wavelet) that satisfies requirements of multiresolution analysis, [1]. From that follows that there exist very wide choice of functions for basis of DWT. The choice of wavelet depends on contents and resolution of image. DWT have some properties, which makes it better choice for image compression than DCT, especially for images on higher resolutions. The entire image is transformed and compressed as a single data object rather than block by block (as in DCT based system) allowing for a uniform distribution of compression error across the entire image. DWT have higher decorrelation and energy compression efficiency so DWT can provide better image quality on higher compression ratios. Localization of wavelet functions, both in time and frequency, gives DWT potentiality for good representation of images with fewer coefficients. DWT represents image on different resolution level. Multiresolution representation is well suited to the properties of Human Visual System (HVS) which gives the possibility for designing different quantizer for each level.

II. BASICS OF WAVELET TRANSFORM

A. Multiresolution Analysis

A function or signal can be viewed as compositions of smooth background and details on top of it. The distinction between the smooth background and details is determined by resolution, that is by the scale below which the details of the signal can not be discerned.

Considering a function $f(t)$ and labeling the resolution level by j the scale below which all fluctuations on that resolution is ignored is $1/2^j$. The function that approximates $f(t)$ is $f_j(t)$. At the next resolution level $j+1$ the details denoted by $d_j(t)$ are included in function $f_{j+1}(t)$, $f_{j+1}(t)= f_j(t)+ d_j(t)$. This procedure can be repeated several times. The function $f(t)$ can be viewed as

$$f(t) = f_j + \sum_{k=j}^{k=\infty} d_k \qquad (1)$$

Similarly, the space of square integrable functions $L^2(R)$ can be viewed as compositions of subspaces $\{W_k\}$ and subspace V_j. $\{W_k\}$ contains details $d_k(t)$. The subspace V_j contains $f_j(t)$approximation of function $f(t)$ on resolution level j.

Requirements of multiresolution analysis are:

1. Subspace V_j must be contained in all subspaces on higher resolutions (2).

$$\ldots \subset V_{-1} \subset V_0 \subset \ldots \subset L^2(R) \qquad (2)$$

2. All square integrable functions must be included at the finest resolution level (3) and only zero function on the coarsest level (4).

$$\overline{\cup_j V_j} = L^2(R) \qquad (3)$$

$$\cap_j V_j = \{0\} \qquad (4)$$

3. All the spaces $\{V_j\}$ are scaled versions of the central space V_0. If $f(t)$ is in space V_j and it contains no details on scales smaller than $1/2^j$, then function $f(2t)$ contains no details on scales smaller than $1/2^{j+1}$ and it is from space V_{j+1} (5).

$$f(t) \in V_j \Leftrightarrow f(2t) \in V_{j+1} \qquad (5)$$

4. If $f(t) \in V_0$, so do its translates by integer k, $\{f(t-k)\}$ (6).

$$f(t) \in V_0 \Rightarrow f(t-k) \in V_0 \qquad (6)$$

5. There exist a function $\phi(t)$, called scaling function, such that $\{\phi(t-k)\}$ is an orthonormal basis of V_0.

B. Scaling Function

The subspaces $\{V_j\}$ form a nested sequence that provides successively better approximation to L^2. Scaling function generates basis functions for each subspace V_j:

$$\phi_{jk}(t) = 2^{j/2}\phi(2^j \cdot t - k), \qquad (7)$$

where coefficient $2^{j/2}$ denotes scale of scaling function and k translation by integer in time. Since $V_0 \subset V_1$, any function from subspace V_0 can be represented with basis functions from V_1. Dilatation equation (8) shows how are related scaling functions, on two successful neighboring resolutions.

$$\phi(t) = \sqrt{2}\sum_k h_k \phi(2t-k) \tag{8}$$

Using the fact that $\{\phi_{jk}(t)\}$ are orthonormal, the coefficients $\{h_k\}$ can be obtained by computing the inner product:

$$h_k = \sqrt{2}\int_{-\infty}^{\infty} \phi(t)\phi(2t-k)dt. \tag{9}$$

These coefficients are called coefficients of lowpass filter or short lowpass filter. Fig. 1 shows example of 4-tap Daubechies scaling function.

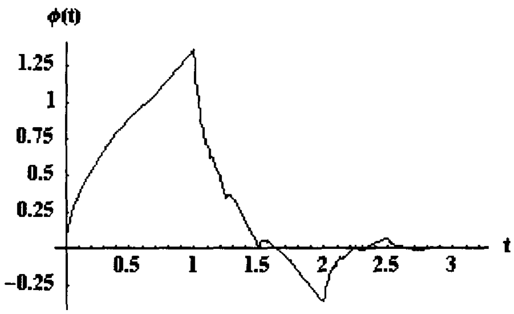


Fig. 1. Example of 4-tap Daubechies scaling function

C. Mother Wavelet

From definition of multiresolution analysis follows that entire space of square integrable functions $L^2(R)$ can be decomposed into orthogonal subspaces $\{W_j\}$ each containing information about details at given resolution. Detail space W_j has orthonormal basis $\{\psi_{jk}(t)\}_k$ where:

$$\psi_{jk}(t) = 2^{j/2}\psi(2^j \cdot t - k) \tag{10}$$

So, $L^2(R)$ has an orthonormal basis $\{\psi_{jk}(t)\}_{jk}$ called wavelet basis. Each wavelet $\psi_{jk}(t)$ is generated by translating and dilating of function $\psi(t)$ called mother wavelet.

Since $\{\psi(t-k)\}$ is in W_0 and $W_0 \subset V_1$, $\psi(t)$ can be represented as superposition of basis functions for V_1.

$$\psi(t) = \sqrt{2}\sum_k g_k \phi(2t-k) \tag{11}$$

Wavelet equation (11) shows how are related mother wavelet and scaling function at the next finer level. Using the fact that $\{\phi_{1k}(t)\}$ are orthonormal, the coefficients $\{g_k\}$ can be obtained by computing the inner product:

$$g_k = \sqrt{2}\int_{-\infty}^{\infty} \psi(t)\phi(2t-k)dt \tag{12}$$

These coefficients are called coefficients of highpass filter or short highpass filter. Coefficients of highpass filter can be calculated from coefficients of lowpass filter using this equation:

$$g_k = (-1)^k h_{1-k}. \tag{13}$$

Fig. 2 shows example of 4-tap Daubechies wavelet function.

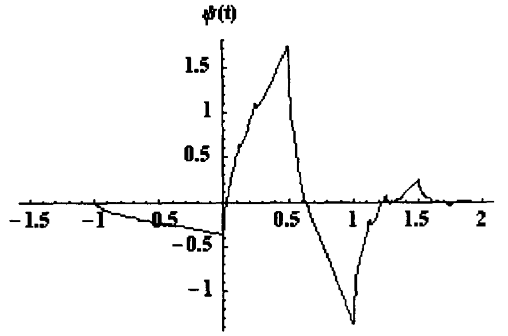


Fig. 2. Example of 4-tap Daubechies wavelet function

D. Discreet Wavelet Transform

Supposing that is input function $f(t)$ only known to the certain resolution level j and that details on the scales smaller than 2^{-j} is ignored the approximation of $f(t)$ on level j is determined by equation:

$$f_j(t) = \sum_k f_k^j \phi_{jk}(t) \tag{14}$$

where $\phi_{jk}(t)$ is a scaling function on resolution level j and translated by integer k and f_k^j are coefficients given by:

$$f_k^j = \int f_j(t)\phi_{jk}(t)dt \tag{15}$$

The function $f_j(t)$ can be uniquely represented by the coefficients $\{f_j^k\}$. Furthermore, $f_j(t)$ can be decomposed into a smooth part $f_{j-1}(t)$ on the next coarser level $j-1$ and details $d_{j-1}(t)$:

$$f_j(t) = f_{j-1}(t) + d_{j-1}(t) = \\ = \sum_k f_k^{j-1}\phi_{j-1,k}(t) + \sum_k d_k^{j-1}\psi_{j-1,k}(t) \tag{16}$$

where $\phi_{jk}(t)$ is a wavelet function on resolution level j and translated by integer k and d_k^j are coefficients given by:

$$d_k^j = \int f_j(t)\psi_{jk}(t)dt \tag{17}$$

Using definition of multiresolution analysis, (9) and (12) expressions for $\{f_j^{k-1}\}$ and $\{d_j^{k-1}\}$ can be derived in terms of $\{f_j^k\}$:

$$f_k^{j-1} = \sum_l h_{l-2k} f_{lk}^j, \tag{18}$$

and

$$d_k^{j-1} = \sum_l g_{l-2k} f_l^j, \tag{19}$$

where l denotes resolution level.

Smooth part of $f_j(t)$ can be further decomposed to smooth part and details on resolution level j-2. Decomposition of input function can be repeated until the coarsest level j_0 is reached.

$$f_j(t) = \sum_{l=j_0}^{j-1}\left(\sum_k d_k^l \psi_{lk}(t)\right) + \sum_k f_k^{j_0} \phi_{j_0 lk}(t) \tag{20}$$

Equation (20) shows decomposition of input function $f_j(t)$ to average part or smooth background and details on $j-1-j_0$ resolution levels.

E. Two dimensional Discrete Wavelet Transform

Discrete Wavelet Transform for two-dimensional signal, or in our case images, can be derived from one-dimensional DWT. Easiest way for obtaining scaling and wavelet function for two-dimensions is by multiplying two one-dimensional functions.

Scaling function for 2-D DWT can be obtained by multiplying two 1-D scaling functions (21). Generally different scaling functions can be used for each direction but in practice those functions are in most cases the same.

$$\phi(x,y) = \phi(x)\,\phi(y) \tag{21}$$

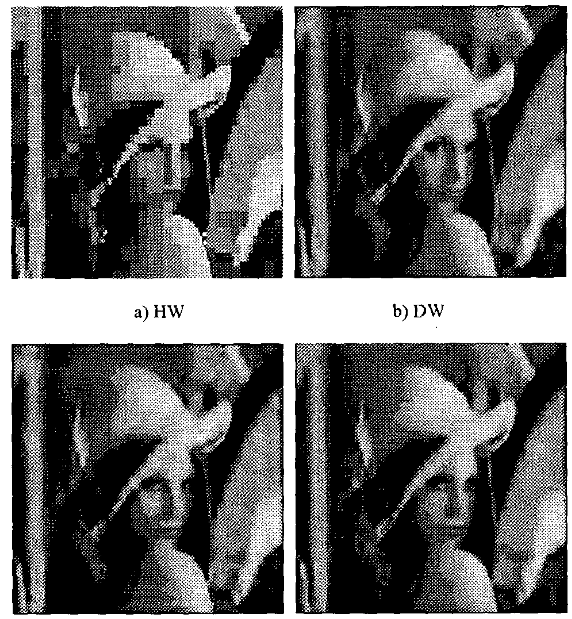
Wavelet functions for two-dimension DWT can be obtained by multiplying two wavelet functions or wavelet and scaling function for one-dimensional analysis. From that follows that for 2-D case there exist three wavelet functions that analyses details in horizontal (22), vertical (23) and diagonal (24) direction.

$$\psi^{(I)}(x,y) = \psi(x)\,\phi(y) \tag{22}$$
$$\psi^{(II)}(x,y) = \phi(x)\,\psi(y) \tag{23}$$
$$\psi^{(III)}(x,y) = \psi(x)\,\psi(y) \tag{24}$$

II. IMAGE COMPRESSION BASED ON DWT

Choice of wavelet function is crucial for coding performance in image compression. This choice should be adjusted to image content. The coding performance for images with high spectral activity is fairly insensitive to choice of wavelet basis (for example test image Baboon), [2]. On the other hand, coding performance for images with moderate spectral activity (for example test image "Lena") are more sensitive to choice of wavelet basis. From that follow, that the best way for choosing of wavelet function is to select optimal basis for images with moderate spectral activity. This wavelet function will give satisfying results for other types of images.

In our experiment four types of wavelet families are examined: Haar (HW), Daubechies Least Asymmetric (DW), Coiflet (CW), and Biorthogonal Spline (BW). Daubechies and Coiflet wavelets are families of orthogonal wavelets that are compactly supported. Compactly supported wavelets correspond to finite impulse response (FIR) filters and thus lead to efficient implementations, [8]. A major disadvantage of these wavelets is their asymmetry, which can cause artifacts at borders of the wavelet subbands. Symmetry in wavelets can be obtained only if we are willing to give up either compact support or orthogonality of wavelet (except for Haar wavelet, which is orthogonal, compactly supported and symmetric). The example of noncompactly supported but symmetric orthogonal wavelet is Meyer wavelet family. The use of Meyer wavelets adds computational burden and can not provide efficient implementation. If we want both symmetry and compact support in wavelets, we should relax the orthogonality condition and allow nonorthogonal wavelet functions. The example is the family of biorthogonal spline wavelets that contains compactly supported and symmetric wavelets.



a) HW b) DW c) CW d) BW

Fig. 3. Comparison of wavelet functions for image "Lena" (256×256) and compression ratio 100:1

Each family can be parameterized by an integer that is proportional to the length of wavelet filter. For compactly supported wavelets, the length of a wavelet filter is related to the degree of smoothness of the wavelet and can affect the coding performance. In our examples the length of wavelet filter is determined by number of taps. Wavelet functions are chosen according to coding performance for image "Lena" with dimensions of 256×256 pixels. The image is coded using different wavelet functions from each wavelet family. Image quality is measured using Peak Signal to Noise Ratio (PSNR) which is defined as:

$$PSNR = 20\log\frac{2^n - 1}{RMS} \tag{25}$$

Root Mean Square Error (RMS) represents difference between original image x_i and reconstructed image x'_i:

$$RMS = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - x'_i)^2} \tag{26}$$

Fig. 3 shows the compression results for wavelet functions that are the best suited to image "Lena" from each wavelet family (2 taps and 8 decompositions for HW, 10 taps and 5 decompositions for DW, 12 taps and 4 decompositions for CW, 4 tap and 6 decompositions for BW). The PSNR results for different wavelet functions and compression ratios are shown in Table I. According to PSNR measurements, the results for higher compression ratios are comparable for DW, CW and BW, but the visual quality is best for BW.

TABLE I
PSNR IN (dB) FOR DIFFERENT WAVELET FUNCTIONS AND COMPRESSION RATIOS (TEST IMAGE "LENA")

Wavelets	Compression ratio			
	5:1	15:1	50:1	100:1
HW	36.01	28.75	23.91	21.86
10-tap DW	37.04	29.33	24.46	22.19
12-tap CW	37.40	29.73	24.63	22.43
4-tap BW	37.99	29.99	24.59	22.33

To examine the influence of the length of a wavelet filter to image quality, we changed the number of taps (2, 10 and 20) in Daubechies Least Asymmetric Wavelet. PSNR values for 2 and 10 taps are contained in Table 1 (Haar wavelet can be seen as Daubechies wavelet with 2 taps). PSNR values for 20 taps and compression ratios 5:1, 15:1, 50:1 and 100:1 are 36.97 dB, 29.01 dB, 24.56 dB and 22.28 dB respectively. Larger number of taps does not imply better PSNR and visual picture quality. Number of decompositions in these examples is chosen to be optimal for corresponding wavelet (8 for 2-tap DW, 5 for 10-tap DW and 4 for 20-tap DW).

Two-dimensional DWT decompose input matrix of image data with dimensions of N×N, to matrix that represent image on the next coarser level, called average part and three matrices of details for each horizontal, vertical and diagonal direction. Those new matrices have dimensions of N/2×N/2. On the next step average part can be considered as an input matrix and procedure of decomposition can be repeated. The number of decompositions is limited by dimensions of original image that is by the dimensions of average part of transformed image that must contain at least one coefficient.

Fig. 4 shows decomposition of image "Lena" with dimensions of 256×256 pixels with 8 bit per pixel to average part and details on each resolution. In this example is used 10-tap Daubechies least asymmetric wavelets with four decompositions. The 4th resolution level denotes the coarsest level. This figure shows that matrices on higher resolution levels contain information of smaller details.

The quality of compressed image depends on number of decompositions because HVS is less sensitive to removal of smaller details. Fig. 5 shows comparison of reconstructed image "Lena" (256×256 pixels) for 1, 2, 3 and 4 decompositions with compression ratio 50:1. It can be seen that image quality is better for higher number of decompositions.

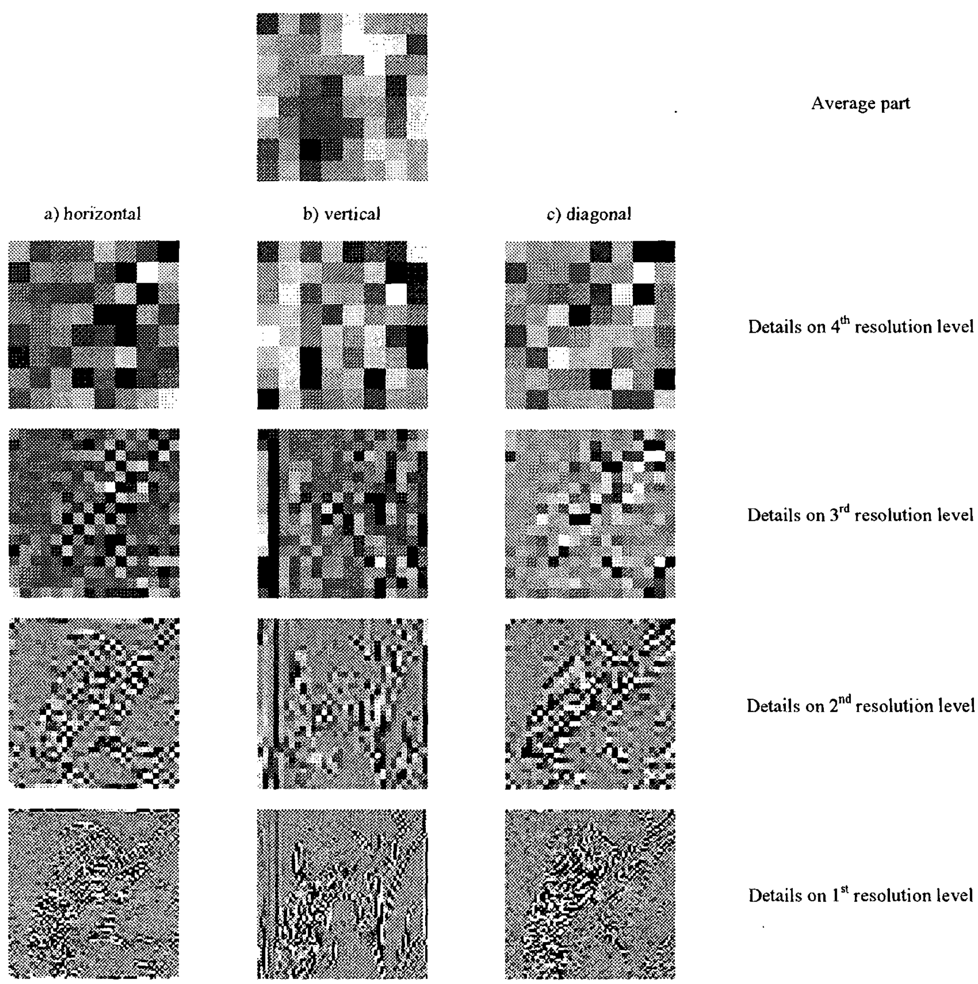
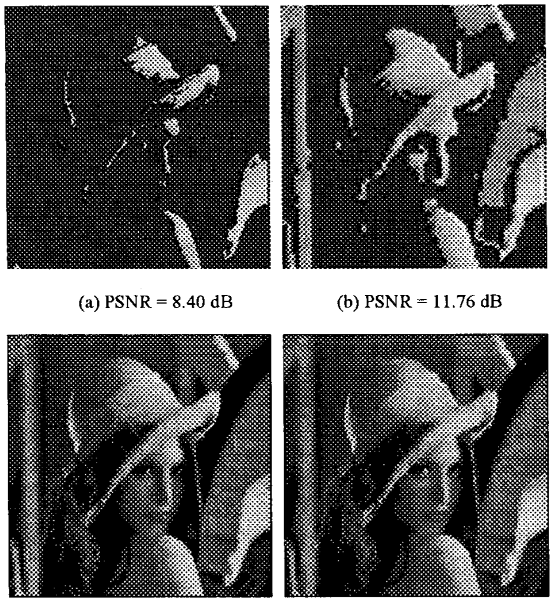


Fig. 4. Decomposition of image "Lena" using 10-tap Daubechies least asymmetric wavelet and four decompositions



(a) PSNR = 8.40 dB (b) PSNR = 11.76 dB

(c) PSNR = 23.39 dB (d) PSNR = 24.40 dB

Fig. 5. Reconstructed image Lena: (a) 1, (b) 2, (c) 3, and (d) 4 decompositions (compression ratio is 50:1)

III. COMPRESSION RESULTS

To achieve higher compression ratios wavelet coefficients can be requantized. Requantization can be applied, by uniform or non-uniform quantization. The fact that DWT represents image on different resolutions gives possibility for designing separate quantizers for each resolution level. The design of each quantizer can be determined from statistics of wavelet coefficients along the different scales [3]. Allocation of bits for each scale can be obtained according to characteristics of Human Visual System, as in [4].

After requantization coding of requantized coefficients can be applied. The aim of coder-decoder pair, or codec is lossles compression of quantized coefficients. The design of codec is usually compromise between requirements for execution speed, available bandwidth and quality of reconstructed image. Run-length coding of zeroes is appropriate for applications that require fast execution. Run-length coded values can be encoded using fixed-length or variable-length codewords. For applications that require best possible quality of reconstructed image, the techniques such as zerotree encoding is better choice, [6]. Zerotree coder gives better image quality than zero run-length coder but the execution time is several times longer.

After decomposing image and representing it with wavelet coefficients, compression can be performed by ignoring all coefficients below some threshold. The different thresholds can be set for each resolution level and also for each decomposition direction: horizontal, vertical or diagonal. After that, coefficients can be requantized. Each resolution has different meaning for subjective quality of reconstructed image. Consequently the best method for requantization is to design different quantizer for each resolution [4].

Comparison of PSNR of image "Lena" for standard JPEG [5] and DWT using 4-tap BW is shown in Fig. 6. Compression results for JPEG are taken from [9]. For compression ratios below 20:1 JPEG gives similar results as DWT. For higher compression ratios (higher than 30:1) quality of images compressed using DWT slowly degrades while quality of standard JPEG compressed images deteriorates rapidly. The compression performance of DWT is superior to that of JPEG and the visual quality of reconstructed images is better even if the PSNR are the same. There are noticeable blocking artifacts in the JPEG images. Fig. 7 shows visual quality for JPEG and DWT compressed images with the same PSNR (26 dB). The comparison demonstrates that even for relatively high compression ratios (>50:1) DWT based compression gives good results according to both visual quality and PSNR (see Table I).

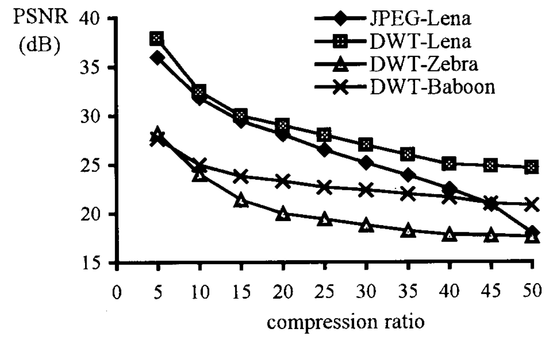
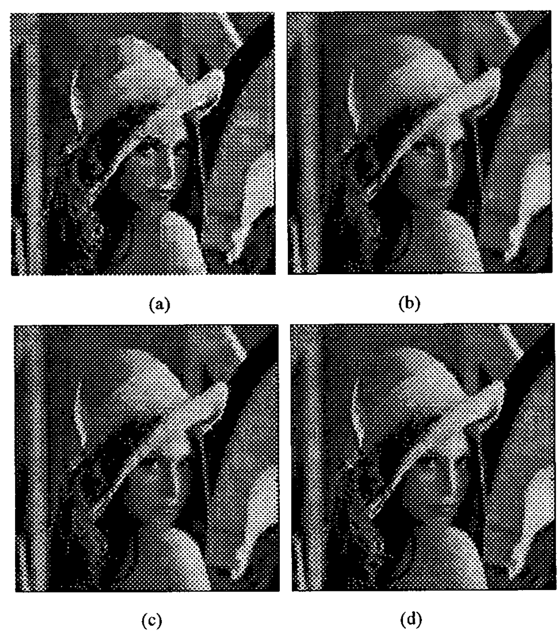


Fig. 6. Comparison of standard JPEG and DWT (4-tap BW) compression



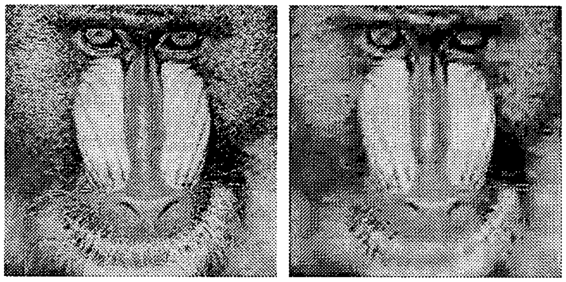
(a) (b)

(c) (d)

Fig. 7.Compression results of image "Lena" using (a) JPEG, (b) DW, (c) CW, (d) BW; (PSNR = 26 dB)

No single image compression algorithm can be expected to work well for all classes of images. Quality of reconstructed image depends on the content of image [10, 11]. Therefore Fig. 6 contains PSNR values for test images Zebra and Baboon compressed using 4-tap BW.

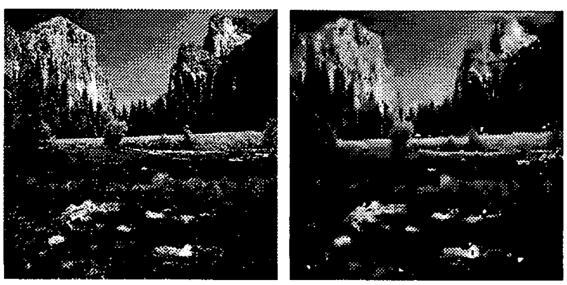
PSNR of "Lena" image is through all compression ratios for about 3 dB higher than PSNR for "Zebra". For other wavelets these relations can be changed. Visual image quality for compression ratios of 5:1 and 50:1 applied to the images Baboon and Zebra are compared in Fig. 8.



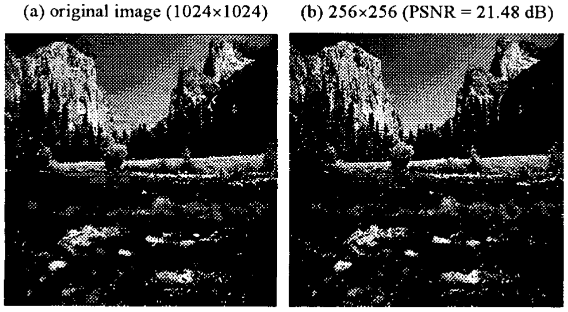
(a) PSNR = 27.71 dB (b) PSNR = 21.67 dB

(c) PSNR = 28.23 dB (d) PSNR = 17.55 dB

Fig. 8. Comparison of reconstructed images Baboon and Zebra compressed using 4-tap BW and compression ratios of 5:1and 50:1



(a) original image (1024×1024) (b) 256×256 (PSNR = 21.48 dB)



(c) 512×512 (PSNR = 21.70 dB) (d) 1024×1024 (PSNR = 23.93 dB)

Fig. 9. Reconstructed image "River" with dimensions 256×256, 512×512 and 1024×1024 pixels for compression ratio 1:50

Quality of reconstructed image depends on dimensions of original image. This follows from fact that DWT gives better results for higher number of decompositions. For larger images more decompositions can be provided. Comparison of visual image quality and PSNR for image "River", with dimensions 256×256, 512×512 and 1024×1024 pixels, compression ratio of 50:1 and 4-tap BW is shown in Fig. 9. Visual quality of larger images is much better than quality of small images while PSNR shows the similar values for all three resolutions.

IV. CONCLUSION

In this article we have provided the basics of wavelet transform and comparisons of different wavelets used in image compression system. Although JPEG processing speed and compression ratio are good there are noticeable blocking artifact at high compression ratios. However, there are no blocking effects at all in reconstructed images by wavelet-based methods. Our very simple and fast compression scheme based on DWT provides better results than standard JPEG especially for higher compression ratios. Researches of possibilities of wavelets for image compression has made great progress in the last five years so that compression schemes based on wavelets have already begun to appear in some software and hardware systems.

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