IMAGE ANALYSER - EDUCATIONAL TOOL

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Abstract: *Statistical and frequency characterisation of image is an important fact for image compression systems. Students specialising in image and video system engineering need to know why these characteristics are important in their field of interest and to understand the influence of these characteristics on image quality. Therefore, we have developed educational software, called ImAn - Image Analyser, that helps students to analyse statistical and frequency characteristics in still image compression systems, that will be presented in this paper.*

Key words: *Image Analysis, Statistical Characteristics, Frequency Characteristics, JPEG, SPIHT, JPEG2000*

1. INTRODUCTION

Image compression for both still and moving images is an extremely important area of investigation, with numerous applications to videoconferencing, interactive education, home entertainment, and potential applications to earth observation, medical imaging, digital libraries, and many other areas. Two compression methods are possible: lossy and lossless. The lossless method keeps all the information, and applies the compression to that information thus enabling the perfect reconstruction. Lossy compression methods are based on removal of statistical and subjective redundancies in images and can achieve much higher compression. By examining these redundancies which can be analysed using statistical and frequency image properties one can determine which type of compression is most suitable for particular image and particular application, and what quality can be expected. Frequency properties give the best insight in subjective redundancy. Subjective redundancy is based on Human Visual System (HVS). Human perception of noise in image is a function of the spatial frequency. More noise can be tolerated at high spatial frequencies. These frequency components can be reduced or removed in a way that is least visible to the viewer. Due to the complexity of HVS, subjective redundancy is hard measurable, but it can be analysed using several statistic and frequency properties of image. The quality of compressed image is measured objectively mostly by PSNR. The values of these quality measures usually can be brought to the relation with properties of the original image, so the information on those properties can be very useful in compression applications.

In this paper we present ImAn - Image Analyser, educational tool for image analysis that we developed. ImAn can help students to learn and understand how to perform statistical and frequency image analysis and what kind of characteristics is important for image analysis. According to the results, students can study correlation between image content and image characteristics, and to analyse reconstructed images after compression in order to evaluate image compression system.

2. STATISTICAL AND FREQUENCY MEASURES OF IMAGES

For purpose of formality, we say that the image is represented with matrix **A**, whose elements represent set *a*, and *a(m,n)* is intensity or luminance value at spatial coordinates *m,n* (*m* is the row, *n* is the column). Some basic properties of image pixels are: $\bar{a}$ - mean value, <*a*> - range of values (min-max), χ_a^2 - mean square value, σ_a - standard deviation (variance), [1]. From these properties some simple conclusions can be derived that are not much relevant to compression applications. Probability density function - *pdf* [1], and zero-order entropy - H_0 ($\log_2$ of number of different values of pixels in image, or average number of bits required to represent one pixel) [2] indicate the type of image. Real world images have continuous *pdf* and high H_0, unlike the most of the artificial images, which have discrete *pdf* and small number of pixel values (low H_0). From these properties, the most appropriate quantization or entropy coding method can also be determined. First order entropy H_1 [2] is a measure of theoretical compression limit for lossless compression method that treats each pixel independently. The real limit for lossless compression is real entropy of image, which is practically incomputable, but higher order entropies can give close approximation. The relation for second order entropy is $H_2 = - p(i, j)\cdot\log p(i, j)$, where $p(i, j)$ denotes the probability that a pixel has value *i* while its neighbouring pixel (in horizontal, vertical or diagonal direction) has value *j* [3]. Presented relation gives second-order entropy or conditional entropy. It is also an average number of bits needed for two pixels, while for one pixel the given number must be divided by two, and we used second order entropy in that one-pixel averaged form. For better insight in dependencies between pixels in image, autocorrelation function can be used, and it can be evaluated separately for rows and columns. We use its normalised autocovariance form (here for rows):

$$\rho_R(k) = \frac{1}{M}\sum_{m=0}^{M-1}\frac{1}{\sigma_m^2}\left(\frac{1}{N-|k|}\sum_{n=0}^{N-|k|-1}\left(a(m,n)-\bar{a}_m\right)\cdot\left(a(m,n+|k|)-\bar{a}_m\right)\right), \qquad (1)$$

where *N* denotes number of pixels in row, *M* number of rows, $\bar{a}_m$ and σ_m mean value and variance of pixels in *m*-th row. Autocorrelation of all image pixels in general can be deduced from a several first values of this function.

Next step is frequency analysis of image. After 2D discrete Fourier transformation of image,

$$\theta(k,l) = \frac{1}{\sqrt{MN}}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}a(m,n)e^{-j\frac{2\pi km}{M}}e^{-j\frac{2\pi ln}{N}}, \qquad (2)$$

coefficients $\theta(k,l)$ lead to power spectral density $psd(k,l) = |\theta(k,l)|^2$, where *k* and *l* are spatial coordinates. For more appropriate investigation of image frequency spectrum, one-dimensional *psd* (1D-*psd*) is derived by computing the mean powers in 2D frequency bands of size $B(\Delta t)$. $B(\Delta t)$ is ring with centre in origin of frequency plane, and is determined in continuous domain by axis frequencies ω_1 and ω_2, which defines spatial frequency $\omega = \sqrt{\omega_1^2+\omega_2^2}$. In discrete form that is $t = \sqrt{k^2+l^2}$. One-dimensional *psd* is the mean power in all such rings:

$$\bar{P}\left(\Delta t = t_1 - t_2\right)_{t_1,t_2} = \frac{1}{B(\Delta t)_{t_1 t_2}}\sum_{t_1 \le t < t_2} psd(k,l). \qquad (3)$$

The shape of *psd* can be described by a spectral flatness measure - *SFM* [1]:

$$SFM(\Theta) = \frac{\left[\prod_{k=0}^{M-1}\prod_{l=0}^{N-1}\left|\theta(k,l)\right|^2\right]^{\frac{1}{MN}}}{\frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1}\left|\theta(k,l)\right|^2}. \tag{4}$$

Totally flat spectrum has *SFM* of 1 and is related to completely unpredictable image (e.g., image that have all pixels with same values, except one pixel with different value), while lower *SFM* indicate more predictable image, or in other words, energy of image is concentrated in fewer coefficients. The conclusion is drawn that if an image has a flat or near-flat spectrum then the quality of any prediction will be poor.

One additional measure is the spatial frequency - *sf* [4], which simply can be described as a mean difference between neighbouring pixels. This definition of frequency in the spatial domain indicates the overall activity in an image. *sf* is defined as:

$$sf = \sqrt{R^2 + C^2}\,,\ R = \sqrt{\frac{1}{MN}\sum_{j=1}^{M}\sum_{k=2}^{N}\left(x_{j,k} - x_{j,k-1}\right)^2}\,,\ C = \sqrt{\frac{1}{MN}\sum_{k=1}^{N}\sum_{j=2}^{M}\left(x_{j,k} - x_{j-1,k}\right)^2}\,, \tag{5}$$

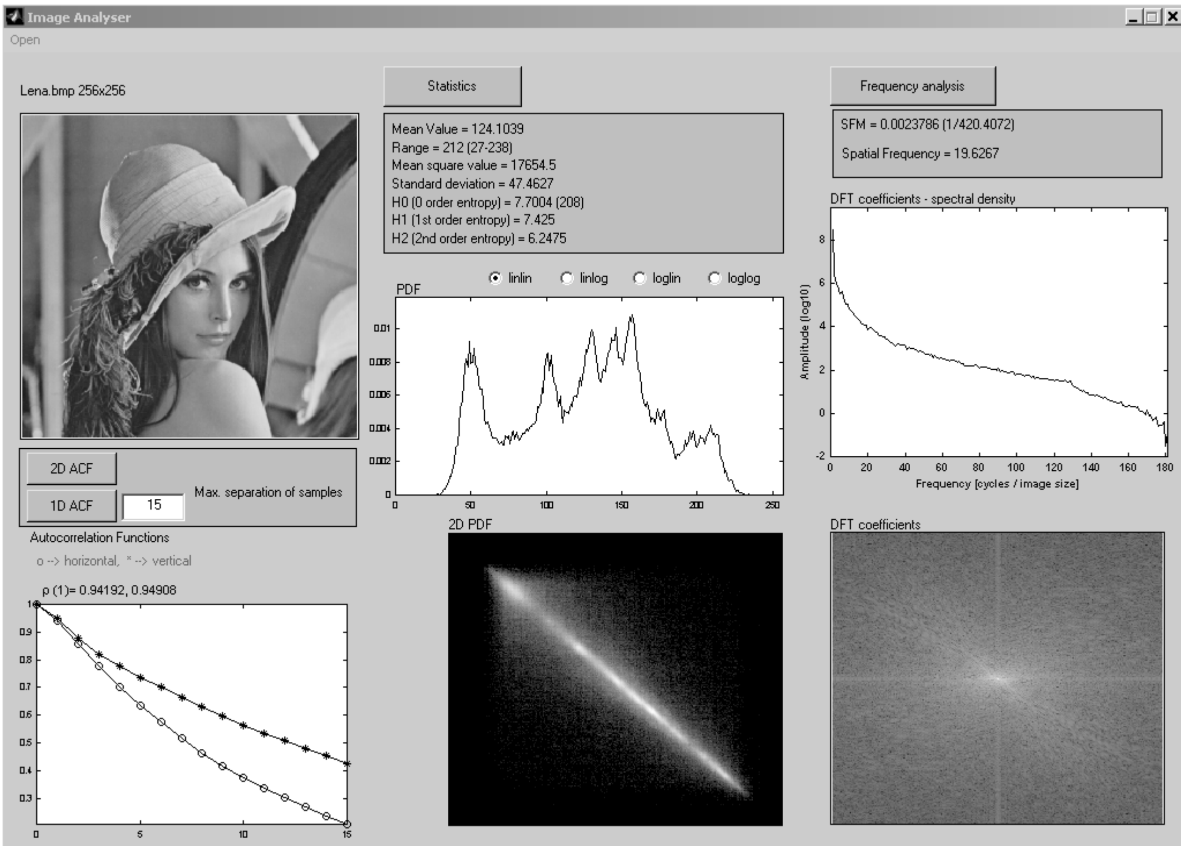


Fig. 1 ImAn window

2. PROGRAM DESCRIPTION

Image Analyser - ImAn, is an educational tool for examining the statistical properties and frequency analysis of images. Fig. 1 shows typical ImAn window with previously performed all evaluations of image properties (the used image is "Bridge"). Input image students can load in the left ImAn frame. Below input image, autocorrelation functions for rows and columns and their values for desired number of sample separation can be displayed. In the middle column of the window, other statistical characteristics are displayed. They include few numerical measures, histogram (*pdf*) and 2D histogram (H_2) plots for chosen image. Student may choose linear or logarithmic display of histogram axes. Frequency analysis, located in the right part of window, can be done by applying DFT on input image. One-dimensional image frequency analysis and SFM value evaluated from DFT coefficients are also displayed.

3. RESULTS

Table 1 presents statistical values that students can see using few artificial and natural images. Image "Noise" is array of random number with values from 0 to 255, and image "Stripes" is a basically two-dimensional square wave. The results in table show typical properties of natural images ("Baboon", "Lena", "Bird") which are high correlation between pixels, high entropy, relatively high difference between first and second order entropies (H_1-H_2), and low frequency characteristic. Artificial images ("Stripes", "Text") have low entropy and higher spectral characteristic. Image "Stripes" has *SFM* of zero and is totally predictable. Image "Noise" shows noise signal properties: no correlation, high entropy and high frequency characteristic.

Table 1. Statistical values of a few input images

Input Image	$\overline{a}$	σ_a	$\rho_C(1)$	$\rho_R(1)$	H_0	H_1	H_2	*SFM*	*sf*
Noise	127.9	74.1	-0.004	-0.002	8.00	7.997	7.901	0.1423	147.77
Stripes	127.5	127.5	0.75	0.75	1.00	1.00	0.77	0	127.25
Text	214.0	72.9	0.62	0.46	4.00	1.72	1.61	0.0198	98.5
Baboon	129.4	42.1	0.59	0.67	7.76	7.35	7.02	0.0199	47.13
Lena	124.1	47.5	0.95	0.94	7.70	7.43	6.25	0.0024	19.63
Bird	125.1	45.8	0.98	0.99	7.65	7.22	5.51	0.0003	9.9

Fig. 2 presents 1D-*psd* for three input images presented in Fig. 3 that have various spectral characteristics (note that the amplitude axis is logarithmic). Students can see that the *psd*-shape is strongly related to the efficiency of compression method that uses subband coding and exploits the properties of Human Visual System (HSV). While it may seem that compression of image with higher frequency characteristic (e.g. "Baboon") would give certainly worse results then one with distinctive low-frequency characteristic (e.g. "Lena"), it can be only said for objective measures (like *PSNR*), while real subjective success actually depends on a case.

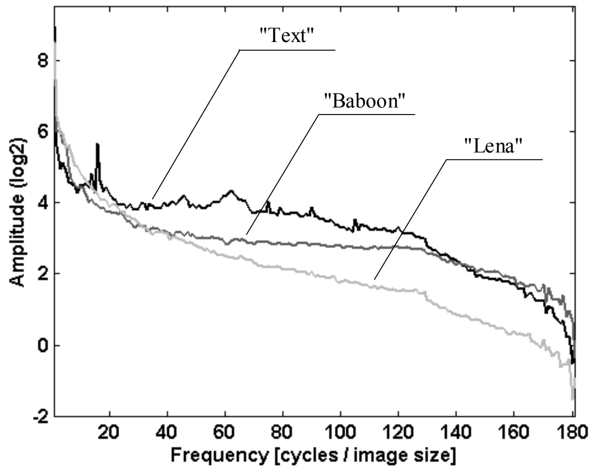
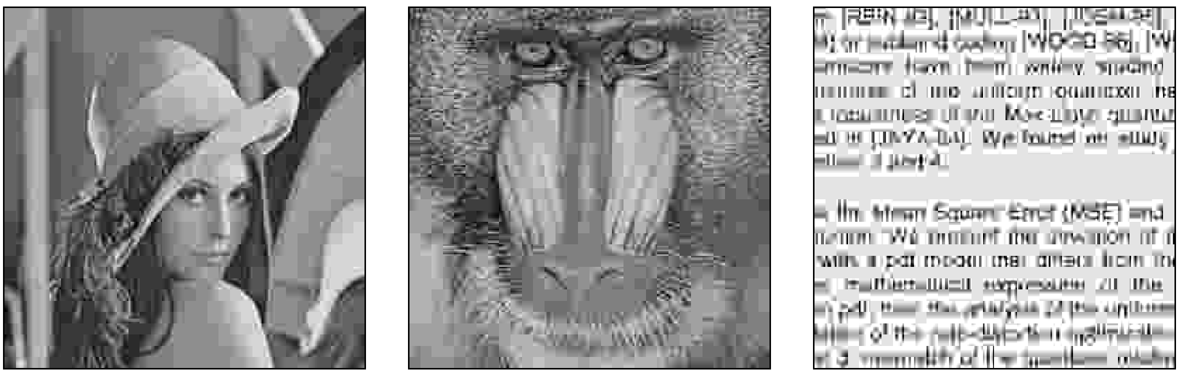


Fig. 2 Spectral characteristics

For example, we performed JPEG image compression on three test images to the rate of 0.3 bpp and results are presented in Fig. 3. Fig. 3(b) has much lower *PSNR* than Fig. 3(a) but it has similar visual quality as Fig. 3(a). Reason for that is that artifacts introduced in Fig. 3(a) have more impact on HVS while large error introduced in Fig. 3(b) is practically irrelevant (mostly in baboons fur, which have noise-like properties). On the other hand, in Fig. 3(c) high portion of image spectrum carries crucial information what results in much poorer visual quality.



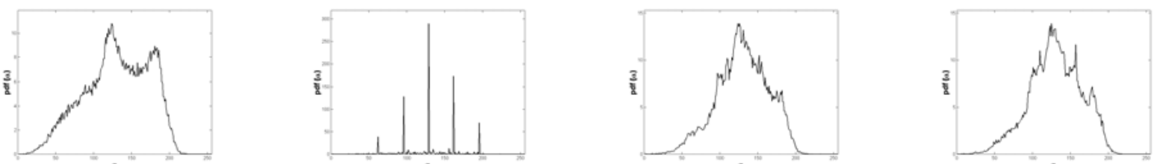
(a) "Lena" - PSNR=27dB (b) "Baboon" - PSNR=20.4dB (c) "Text" - PSNR=13.1dB

Fig. 3 JPEG compressed images (0.3 bpp)

ImAn can also be used to compare efficiency of different compression methods. In this case, reconstructed image after compression is used as an input image loaded in ImAn. Fig. 4 presents result of 3 compression methods applied to image "Baboon": JPEG, SPIHT and JPEG2000. Fig. 5 shows the histograms (*pdf*), Fig. 6 2D histograms, and Fig. 7 spectral densities of original and compressed images achieved using ImAn.

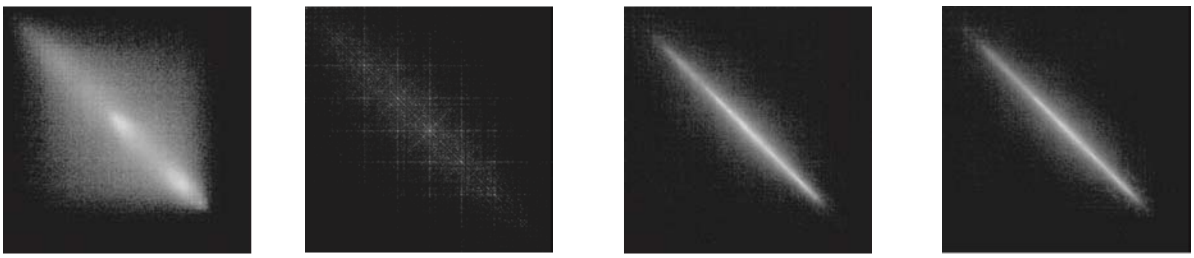
(a) Original (b) JPEG (c) SPIHT (d) JPEG2000

Fig. 4 Image "Baboon" compressed at 0.1 bpp



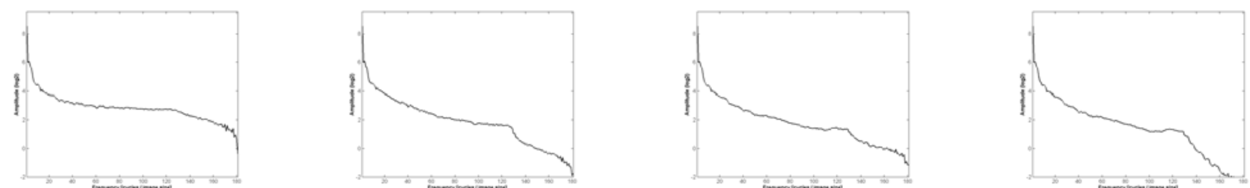
(a) Original (b) JPEG (c) SPIHT (d) JPEG2000

Fig. 5 Histograms (*pdf*)



(a) Original (b) JPEG (c) SPIHT (d) JPEG2000

Fig. 6 2D histograms



(a) Original (b) JPEG (c) SPIHT (d) JPEG2000

Fig. 7 Spectral densities

It can be seen that 1D and 2D histogram of JPEG compressed image clearly show the impact of DC coefficients quantization, and resulting blockiness in image. Histograms of SPIHT and JPEG2000 compressed images show smoothing property of wavelet based image compression. Spectral densities show LF filter behaviour of all three methods.

4. CONCLUSION

We have presented ImAn - Image Analyser, educational software with didactic objective, which combines theory and practice in a very convenient way. In this software it is possible to analyse statistical and frequency image characteristics important for every image compression system. Students can explore the influence of image content to these characteristics and to the image quality. Using ImAn students become fully active participants in the learning process and achieve a much deeper, practical and more permanent understanding of the image processing concepts.

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