An Algorithm for Counting of Intermodulation Products in Multicarrier Broadband Systems

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Abstract **- The paper presents an algorithm for selective counting the number of intermodulation products in multicarrier systems where several signals are transmitted through the system with nonlinear transfer characteristic. The presented formulas show significant reduction in computational effort, compared to the direct counting of intermodulation products, specially if the number of intermodulation products is large. The major assumption used in the derivation of number of beats per channel is that the carriers are equally spaced in frequency.**

I. INTRODUCTION

When several carriers are transmitted through a nonlinear system, a large number of intermodulation (IM) products are generated. Especially in CATV systems with equal channel spacing, various IM products generated in the amplifiers have the same frequency, which results in very high levels of interference at certain frequencies. Frequencies $A \pm B$ and $2A$ introduce the second order IM distortion. Grouped together, second order IM products of $A \pm B$ type form composite second-order beats (CSO). Frequency products due to third order nonlinearity $A \pm B \pm C$, $3A$ and $2A - B$ form third-order beats. Composite triple beat (CTB) distortion is generated from $A \pm B \pm C$ components.

Analysis of the amplitudes of different nonlinear distortion products in modern CATV systems shows that CTB products are the most significant problem in the system. Composite second order products (CSO) are supressed in push pull amplifier configurations, and do not contribute significant to the distortion level when cascading lot of amplifiers.

The strongest and most important third order products occure as a result of interference of three input frequencies $A \pm B \pm C$. These products appear on the carrier frequencies. Third order products of $3A$ have usualy 15.6 dB lower level, and $2A - B$ are 6 dB lower than $A \pm B \pm C$ products, and they do not contribute significant to the amplitude of third order intermodulation products.

II. DERIVATION OF THE NUMBER OF BEATS

The major assumption used in the derivation of number of beats per channel is that all carriers are equally spaced in frequency. The frequency spacing between channels can be any value as long as it is the same between every carrier.

The analysis considers a real CATV system with 27 channels in VHF band. The lowest channel is S2, and the highest is S20 with 7 MHz channel spacing in the frequency range from 111-300 MHz.

The intermodulation beats that fall in the frequency band of a CATV system were calculated by grouping the CTB products in 3 groups, to avoid counting some beats twice. These groups were $A+B-C$, $A-B+C$ and $-A+B+C$. If we presume that $A < B < C$, most products have positive frequencies. Other CTB products such as intermodulation products of $A+B+C$ or $A-B-C$ group fall outside the frequency band of our interest.

To derive the formula for the number of beats, graphical distribution of CTB products per frequency was analysed for a small number of carriers. The first channel in all cases was the channel S 2, with the carrier at 112,25 MHz. The number of beats for each group was represented mathematically, and then combined and summed up to get the final formula for counting the total number of CTB products per channel.

Analyzing the curves of the distribution of CTB products per frequency we can see that for the same number of channels in a system, these three groups have diferrent distribution of CTB products.

We will express the terms for the calculation as:

Z - total number of frequencies at which CTB products are distributed
x - single beat frequency
f_{min} - lowest frequency component with $x = 1$
f_{max} - highest frequency component with $x = Z$
N - total number of channels in the system
M - actual channel of interest
f_d - frequency of the lowest channel ($M = 1$)
f_g - frequency of the highest channel ($M = N$)
f_r - frequency spacing of the channels
(all frequencies are in MHz).

A+B–C group:

Fig. 1. represents the frequency distribution of CTB products for this group with the terms:
$Z = N + (N - 5)$;
$f_{min} = 2f_d - f_g + f_r$;
$f_{max} = f_g - 3f_r$.
The highest number of these CTB products is at the frequency f_M with
$f_M = (f_{min} + f_{max})/2 = f_d - 2f_g$

Lowest channel degraded with these products is the first channel in the system ($M = 1$). Carrier frequency of this channel is much higher than the lowest frequency occupied with CTB products of this group. At the first channel we have $x = (Z + 3)/2$. The highest channel occupied with these products is $M = N - 4$, which is at the frequency f_{max} where we have $x = Z$. Above this channel we do not have any CTB product due to this group. We can express the number of products B_X at single frequency in every channel of our interest as:

$$B_{X(A+B-C)} = [(N-M)^2 - 2(N-M)]/4 \qquad (1)$$

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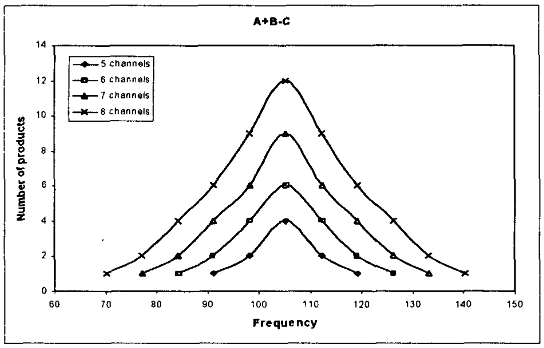

Fig. 1. Distribution of CTB products of A+B–C group

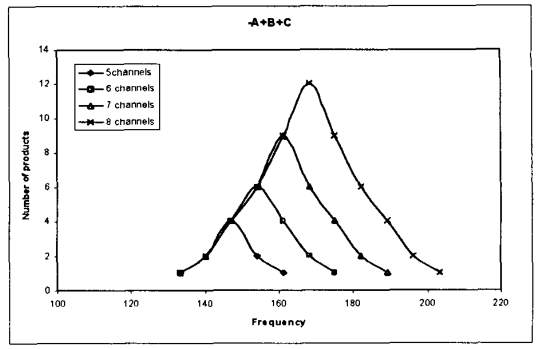

Fig. 3. Distribution of CTB products of –A+B+C group

A–B+C group:

Frequency distribution of CTB beats of this group is shown in Fig. 2. For this group the terms are:

$Z = N - 2$

$f_{min} = f_d + f_r$

$f_{max} = f_g - f_r$

The highest number of products is at the frequency

$f_M = (f_{min} + f_{max})/2 = (f_d + f_g)/2$

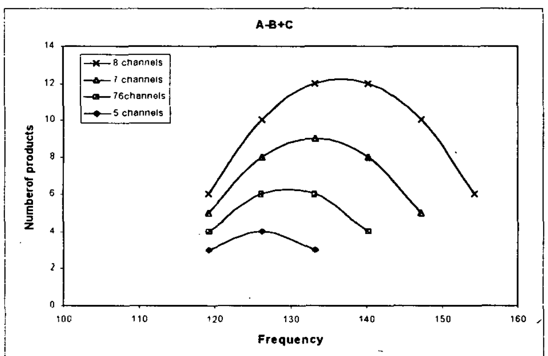

Fig. 2. Distribution of CTB products of A–B+C group

The number of products B_X versus frequency from f_{min} with $x = 1$, to f_{max} with $x = Z$, changes according to the expression $B_X = x\,(N - (x + 1))$. If we want to express this formula in dependence of the actual channel, we change the term x with $M - 1$, as for this group is $x = M - 1$. We can express the number of products B_X at single frequency in any channel of our interest as:

$$B_{X(A\text{-}B+C)} = (M - 1)(N - M) \qquad (2)$$

–A+B+C group:

Frequency distribution of CTB beats of this group is shown in Fig. 3. For this group the terms are:

$Z = N + (N - 5)$

$f_{min} = f_d + 3f_r$

$f_{max} = f_g + (N - 2)f_r$

The highest number of products is at the frequency f_M

$f_M = (f_{min} + f_{max})/2 = f_g + f_r$

The number of products B_X versus frequency from the lowest channel occupied with these products ($M = 4$), at the frequency f_{min} with $x = 1$, to the highest channel ($M = N$) with the value of $x = (Z - 1)/2$ can be expressed as:

$$B_{X(B+C\text{-}A)} = (M^2 - 4M + 3)/4 \qquad (3)$$

III. TOTAL NUMBER OF CTB PRODUCTS

Analysing the curves of the distribution of CTB products we see that all these groups have the same highest number of products B_{Xmax} for the same number of channels. It can be expressed as:

$$B_{Xmax} = (N^2 - 2N)/4 \qquad (4)$$

Summing up the products of those three groups we get the total number of CTB products in every channel of our interest. This can be expressed as:

$$B_{Xsuma} = \frac{(M + N)^2 - 3M^2 + 2M - 6N + 3}{4} \qquad (5)$$

This formula is valid for systems with any number of channels, if the carrier frequencies are equally spaced. It is remarkably accurate when compared to the computer calculations with large numbers of carriers. Fig. 4. shows the frequency distribution curves of CTB products according to (1), (2), (3), and (5) for a 27 channel system.

The curves for $A+B-C$ and $-A+B+C$ products outside of the used frequency band for the transmission, that is the band below the first transmitted channel and above the highest transmitted channel, are not defined with formulas for counting the number of CTB products per channel.

In the 27 channel system there is practically no difference between the calculated and the real number of CTB products, as it is shown in Fig. 5. The total difference between the real number of products and the calculation is less than 0,5 which can be neglected.

IV. AMPLITUDES OF CTB PRODUCTS

In previous discussion only the quantity of beats were analysed. In a well behaved amplifier we can assume that all triple beat products will have the same level. As the carrier frequencies in CATV systems can varie in the order of few kHz from the nominal value, composite triple beat distortion products do not fall exactly at carrier frequencies, but in a narrow range near the carrier frequencies. They are considered as a group, and have a band-spread of about ±15 kHz. The spectrum of this groups of beats is noise-like, because it is made up of many beats.

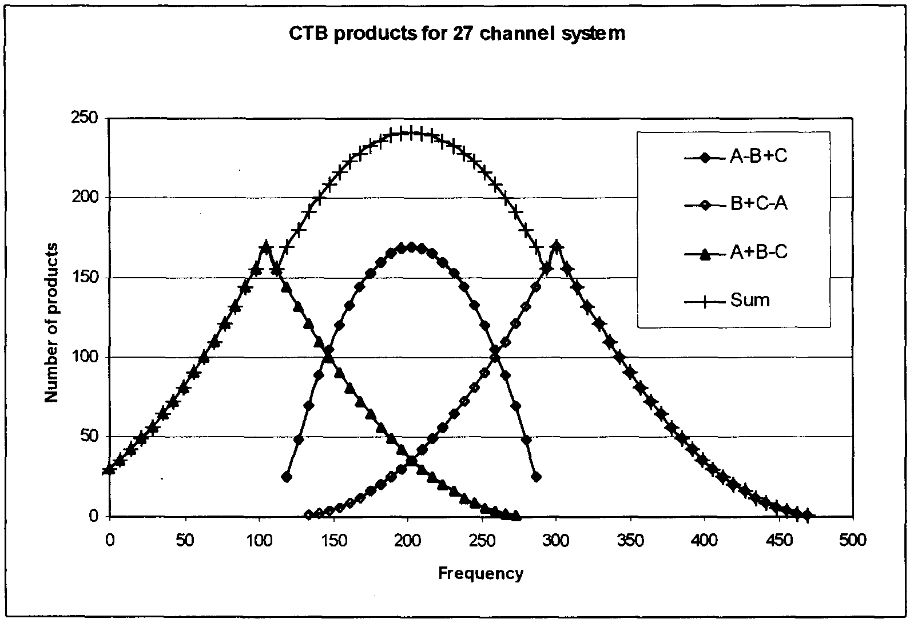

Fig. 4. Distribution of CTB products in a 27 channel system

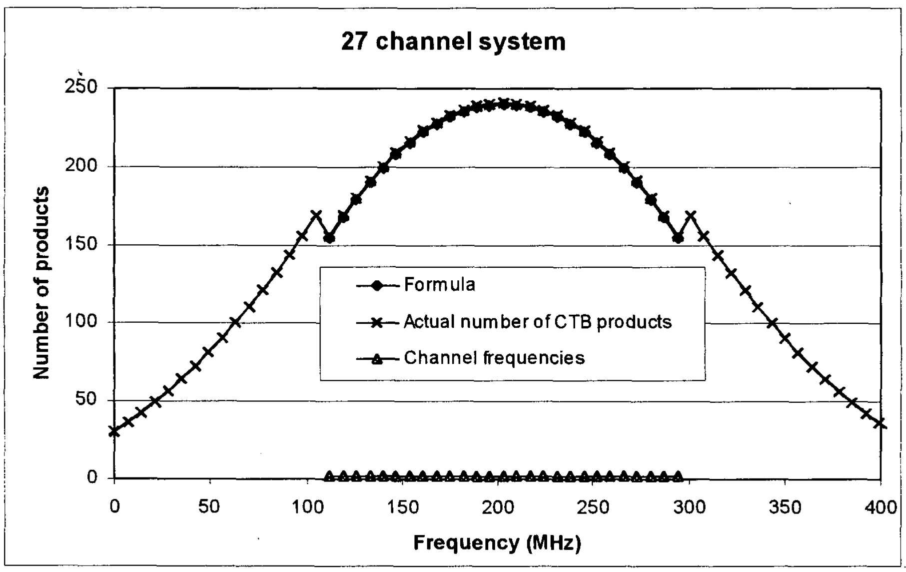

Fig. 5. Comparision of actual number of CTB products with the calculation

The power of the composite of the beats is in general the sum of all the power in the individual beats. Measuring the power of one distortion beat and knowing the number of beats per channel, we can determine the total power of the CTB distortion in the channel of interest.

We can establish that the level of CTB disturbance changes with the total number of channels as:

$$CTB_{total} = CTB_{single} + 10 \log B_{max} \quad (6)$$

where B_{max} represents the total number of CTB products in the worst channel.

As we can see from the curves, the highest number of CTB beats occurs at the middle frequency of the transmission band, so we can write that $M = N/2$ for the worst channel. The value for B_{max} can be expressed as:

$$B_{max} = \frac{3N^2 - 10N + 6}{8} \quad (7)$$

V. CONCLUSION

The presented formulas for counting composite triple beat intermodulation products in a single channel can be used in any system, if carrier frequencies have the same channel spacing. It is very useful for systems where many channels are transmitted through the system with nonlinear transfer characteristic. Direct counting of Composite Triple Beat (CTB) products is very time consuming, and the presented formulas can significantly reduce the computational effort, specially if the number of channels in the system is large. Knowing the amplitude of the single CTB component, and the total number of CTB beats in the particular channel, we can calculate the level of the disturbance for any channel.

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