OFDM Error Floor Based EVM Estimation

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Abstract— **The residual BER – error floor, though useful and widely used metrics for the end-to-end digital radio transmission performance, provides no insight into the error-generating analog impairments (e.g. modulation inaccuracy, power amplifier compression, carrier recovery phase error, or I-Q cross-talk), which, however, are easy to identify by VSA tools such as constellation analysis. As the EVM analysis has become very popular figure of merit in this regard, in this paper, after reviewing crucial advantages of EVM analysis with respect to what we get from BER, we consider estimating EVM from the residual BER assuming that the latter's generally non-linear and non-additive "constituents" are substituted by the equivalent AWGN source producing the same BER (and EVM) degradation. The resulting EVM(BER) curves were verified in the LTE lab to be a very good first approximation of EVM from available residual BER, when (expensive) VSA tools are not available.**

Keywords—BER, OFDM, LTE, time-dispersive channel

I. Introduction

The primary performance measure of a digital radio system is the *Bit-Error-Rate* – BER. However, the Long-Term Evolution (LTE) specifications express the physical layer performance in terms of the *Block-Error-Rate* – BLER, rather than with BER. This is due to rising awareness that, in many practical situations, in-service detecting and counting negative receiver acknowledgements about the successfulness of the data block transfer, relative to the total acknowledgements, which is performed by the *Hybrid Automatic Repeat Request* (HARQ) error control protocol at the link layer, has many advantages over out-of-service BER measurements that presume transmission of pseudorandom binary sequences instead of real traffic.

However, in-service testing may provide inaccurate low BLER values (e.g. 10^{-5}), which can significantly increase the protocol-data-unit retransmission rate of higher-layer protocols (TCP) and so reduce the throughput of the information ("goodput"). Therefore, testing BER remains unavoidable in LTE digital radio products design and manufacturing, and in some instances (e.g. during equipment installation) in the field.

In an AWGN radio channel, as the *Signal-to-Noise-Ratio* (SNR) gets larger, the BER falls down to its irreducible lower limit called residual BER or error floor, which remains constant regardless increasing the signal strength, and presents the 'normal' operating performance of the data link.

In LTE terms, these errors give rise to the related remaining uncorrected block errors determining the related *residual channel* [1]. Specifically, the explicit prediction of the residual BER was proposed for the case of the Orthogonal Frequency Division Multiplexing (OFDM) signal transmission in a small-time-dispersion environment, indoor in particular [1]:

$$BER = \frac{1}{2\sqrt{\pi}} \cdot \sqrt{W^{-}\mathrm{E}\left[\left(\frac{\tau^{-}}{T_s}\right)^2 \Delta s^2_{n/n+1}\right]} + \sqrt{W^{+}\mathrm{E}\left[\left(\frac{\tau^{+}}{T_s}\right)^2 \Delta s^2_{n-1/n}\right]} - \frac{1}{2\sqrt{\pi}} \cdot \frac{\sqrt{W^{-}\mathrm{E}\left[\left(\frac{\tau^{-}}{T_s}\right)^2 \Delta s^2_{n/n+1}\right]} \cdot \sqrt{W^{+}\mathrm{E}\left[\left(\frac{\tau^{+}}{T_s}\right)^2 \Delta s^2_{n-1/n}\right]}}{\sqrt{W^{-}\mathrm{E}\left[\left(\frac{\tau^{-}}{T_s}\right)^2 \Delta s^2_{n/n+1}\right]} + \sqrt{W^{+}\mathrm{E}\left[\left(\frac{\tau^{+}}{T_s}\right)^2 \Delta s^2_{n-1/n}\right]}} \quad (1)$$

where intersymbol interference due to multipath propagation is the dominant impairment. The common channel-dispersion parameters in (1) are: rms delay spreads $E\left[\left(\frac{\tau^{-}}{MT_s}\right)^2\right]$ and $E\left[\left(\frac{\tau^{+}}{MT_s}\right)^2\right]$, normalized to the original symbol interval and distinguished for the advanced and delayed multipath echoes with respect to the chosen sampling instant, respectively, as well as the corresponding composite powers W^{-} and W^{+}, respectively, relative to the total mean power of all echoes. The signal related parameters in (1) are the differences $\Delta\hat{S}_{n/n+1}$ and $\Delta\hat{S}_{n-1/n}$ between the *n*-th and its following (*n*+1)-th OFDM symbol in a row, and between the (*n*-1)-th and *n*-th OFDM symbol, respectively [1].

Nevertheless, although there is no doubt about the significance of the residual BER as the key end-to-end quality of service performance metrics, still it only indicates a problem, but provides no useful clue (such as analog parameter value) on the cause, which would have pinpointed to a specific system component such as transmitter, modulator, oscillator, digital signal processor, transmission path, receiver, or demodulator.

The state-of-the-art vector signal analysis (VSA) [2] definitively provides a number of ways to handle these issues. The LTE lab schematic that we use for the VSA measurements and a typical VSA screen shot are presented in Fig. 1 and Fig. 2, respectively [3].

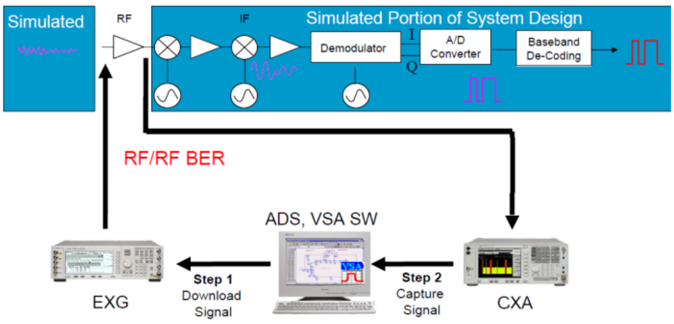

Fig. 1 RF-to-RF BER and VSA testing

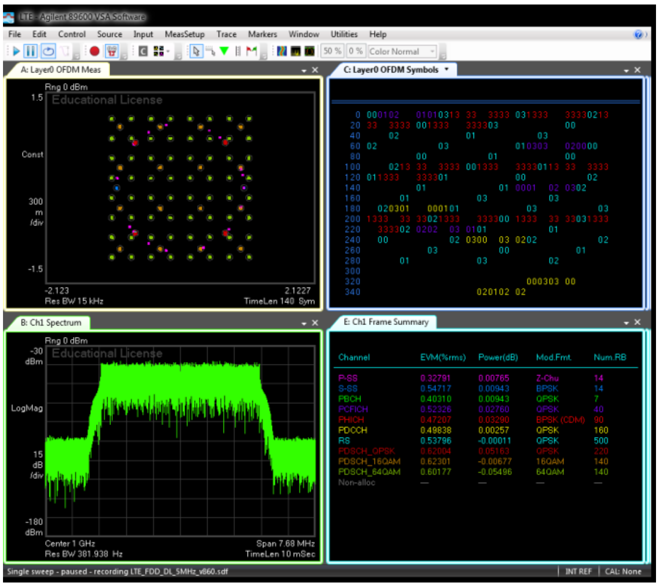

Fig. 2 Typical modulation channel VSA screen shot

Specifically, the polar I vs Q vector diagram presents traces of carrier transitions from symbol to symbol at virtually all points in time, so e.g. providing the information to amplifier designers on the adequacy of bias and loading levels.

While the vector diagram is the best way to view the transition trajectories among states, it can be made to look like a constellation by synchronizing the time base of the analyzer to the symbol clock. The resulting constellation diagram provides carrier amplitude and phase at symbol clock time instances only, and so is an excellent modulation signal analysis tool, enabling visible insight not only into the additive noise level of the received signal, Fig.2, but also about many other qualitative signal characteristics, coming out of particular constellation shapes.

Consequently, imperfect constellation patterns pinpoint to the impairments that combined together determine the residual BER, and include e.g. modulator gain and phase imbalance (usually attributed to the modulator or IF section), power amplifier distortion, carrier recovery circuits or I-Q cross-talk, excessive phase noise in the oscillators, improper filtering, clock jitter etc. [2]. So, these can be easy identified and taken care of.

However, although such analog plots are very useful to identify large impairments, the distortions smaller than 10% of peak values may be difficult to notice. In that case, specific VSA analysis based on Error Vector Magnitude (EVM) measurements is the best option.

Finally, having realized the benefits of the EVM analysis over the pure BER figure, it makes sense to estimate EVM from the residual BER (1), which is the goal of this paper.

In Section II, we review the EVM measurement concept, while in Section III, we focus relationship between the error floor and its belonging EVM and the according EVM estimation from given residual BER. Final conclusions are summarized in Section IV.

II. Error vector vs time and EVM

EVM measurements are sometimes used as an alternative to BER testing, as it provides insight into the modulation quality, specifically with multi-symbol modulation methods such as M-ary Phase-Shift Keying (M-PSK) and M-ary Quadrature Amplitude Modulation (M-QAM) that are widely deployed in wireless local-area networks (WLAN), broadband wireless, and 4G cellular radio systems such as LTE, where M-QAM is combined with OFDM modulation.

As it is presented in the I/Q plane, the Error Vector (EV) is defined as the difference vector between the ideal (or reference) symbol vector and the actual vector assigned to that very symbol, Fig.3.

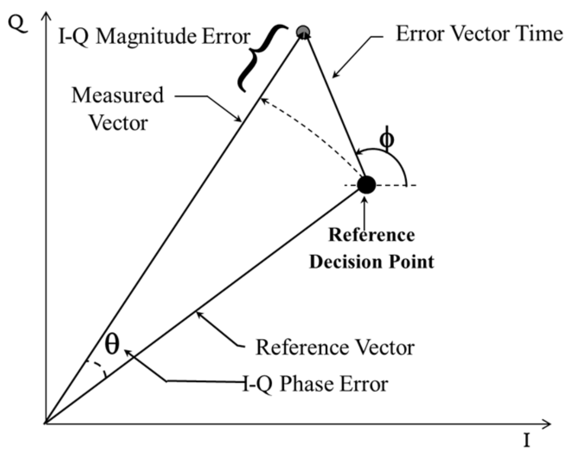

Fig.3 EV vs time (EVT)

With this regard, it is especially useful to measure the EV versus time (EVT), as it was introduced long ago by Hewlett Packard [4]:

$$\mathrm{EVT} = \frac{\sqrt{[\mathrm{I}_{\mathrm{ref}}(t) - \mathrm{I}_{\mathrm{mea}}(t)]^2 + [\mathrm{Q}_{\mathrm{ref}}(t) - \mathrm{Q}_{\mathrm{mea}}(t)]^2}}{\sqrt{\mathrm{I}^2_{\mathrm{ref}}(t) + \mathrm{Q}^2{}_{\mathrm{ref}}(t)}} \quad (2)$$

$$\Phi(t) = Tan^{-1}\frac{(\mathrm{Q}_{\mathrm{meas}} - \mathrm{Q}_{\mathrm{ref}})}{(\mathrm{I}_{\mathrm{meas}} - \mathrm{I}_{\mathrm{ref}})} \quad (3)$$

which references the error to the ideal decision points, so that the residual error at the symbols as well as between symbols can be computed, Fig. 4.

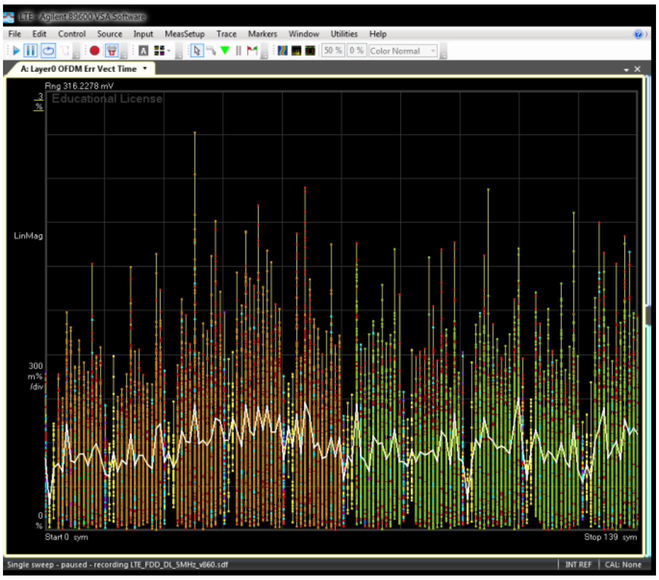

Fig. 4 EVT

The characteristics of these small deviations provides clear differentiation between many types of impairments and so enables assessment of multi-level, multi-phase modulation signals quality based on the measured amplitude and phase distortions that were too small to be visible in the constellation, vector, and eye traces.

Furthermore, averaging (2) along the data sequence provides the rms value of the EVM usually defined in relative terms, i.e. as the ratio of averages of the EV power (P_{error}) to the ideal (reference) symbol vector power (P_{ref}):

$$\mathrm{EVM} = \sqrt{\frac{\frac{1}{N}\cdot\sum_{i=1}^{N}\left\{[\mathrm{I}_{\mathrm{ref,i}}(t) - \mathrm{I}_{\mathrm{mea,i}}(t)]^2 + [\mathrm{Q}_{\mathrm{ref,i}}(t) - \mathrm{Q}_{\mathrm{mea,i}}(t)]^2\right\}}{\frac{1}{N}\cdot\sum_{i=1}^{N}\left[\mathrm{I}^2_{\mathrm{ref,i}}(t) + \mathrm{Q}^2{}_{\mathrm{ref,i}}(t)\right]}} \quad (4)$$

where all squared I and Q components in (4) are properly normalized (so that the total power of any constellation equals unity), enabling that EVM values of various modulation format (16QAM and 64 QAM that can coexist in LTE downlink) can be mutually compared.

The rms EVM is mostly expressed in dBs:

$$\mathrm{EVM\ (dB)} = 10 \log (P_{error}/P_{ref}) \quad (5)$$

or as a percentage:

$$\mathrm{EVM\ (\%)} = \sqrt{(P_{error}/P_{ref})} \times 100 \quad (6)$$

If the symbol errors were caused only by noise, EVM would be equal to SNR at each sample point. However, as it was already mentioned above, other sources of modulation errors exist that are neither additive nor linear, in which case the EVM has been accepted to be an appropriate overall single-number indicator of radio link health.

Typical VSA Error Summary is shown in Fig. 5, where the displayed results include the various EVM values, among them the overall rms, the peak and data only.

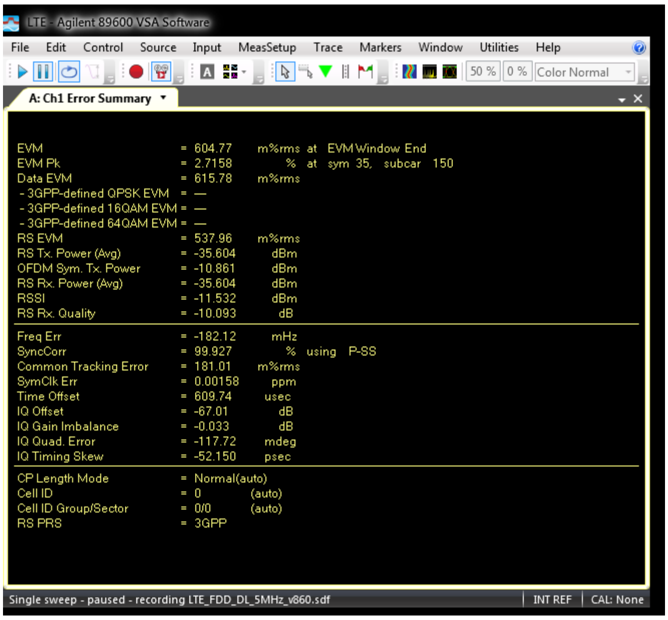

Fig. 5 Error Summary

III. Estimating EVM from residual BER

Finally, having justified the advantages of EVM analysis with regard to what we get from BER, the residual one in particular, we consider useful to estimate EVM from BER.

Now let us assume that all residual BER (and the related EVM) “constituents” are substituted by the equivalent additive Gaussian noise source producing the same BER (and EVM) degradation [5].

With this regard, let us review the well-known BER expression for the M-QAM signal transmission over the AWGN channel with [6]:

$$BER = \frac{4}{\log_2 M} \cdot Q\left(\sqrt{\frac{3\frac{E_b}{N_0} \cdot \log_2 M}{M-1}}\right) \tag{7}$$

where E_b and N_0 are energy of bit and noise spectral density, respectively and Q denotes the Gaussian tail function. The familiar very steep ("waterfall") curves, shown in Fig. 6 visually reflect the threshold effect at the digital radio receiver.

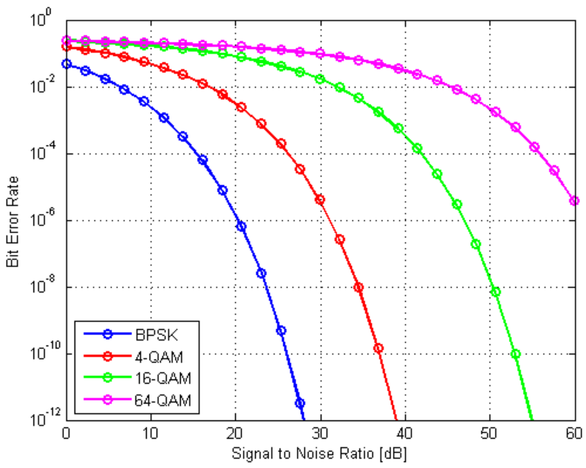

Fig. 6 Waterfall BER(SNR) curves (for Nyquist BW)

Taking into account that SNR can be expressed as:

$$\frac{E_s}{N_0} = SNR = \frac{E_b}{N_0} \cdot \log_2 M \tag{8}$$

where Es is the energy of symbol, it follows that:

$$\frac{E_b}{N_0} = \frac{SNR}{\log_2 M} \tag{9}$$

Furthermore, substituting [7]:

$$SNR \approx \frac{1}{EVM^2} \tag{10}$$

into (7), it can be rewritten as:

$$BER = \frac{4}{\log_2 M} \cdot Q\left(\sqrt{\frac{3}{EVM^2 \cdot (M-1)}}\right) \tag{11}$$

From (11) it follows:

$$\frac{BER \cdot \log_2 M}{4} = Q\left(\sqrt{\frac{3}{EVM^2 \cdot (M-1)}}\right)$$

$$\frac{1}{EVM^2} = \frac{(M-1) \cdot \left[Q^{-1}\left(\frac{BER \cdot \log_2 M}{4}\right)\right]^2}{3}$$

and finally:

$$EVM(BER) = \sqrt{\frac{3}{M-1}} \cdot \frac{1}{\left[Q^{-1}\left(\frac{BER \cdot \log_2 M}{4}\right)\right]} \tag{12}$$

where Q^{-1} is the inverse of the Gaussian tail function. The plot EVM(BER) is given in Fig. 7.

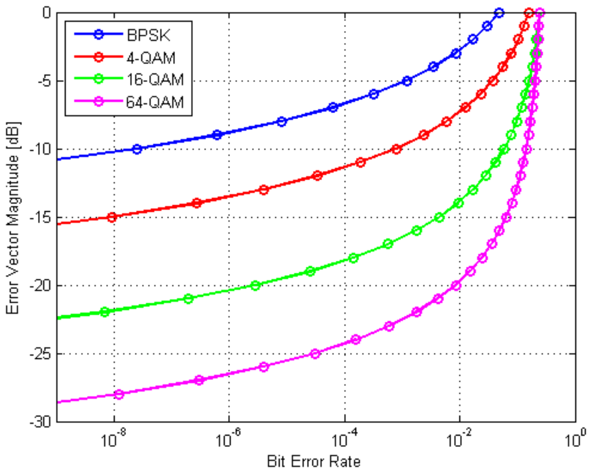

Fig. 7 EVM vs Residual BER

In order to verify (12), we conducted embedded coded BER/BLER tests on the LTE downlink channel (PDSCH 1 (UE 1) with high SNR [3]. The reference channels complied with TS 36.101, and the faded one to the definition in Annex B of TS 36.101, with no HARQ error control deployed. Specifically, we focused the Extended Pedestrian A (EPA) channel delay profile model.

The obtained average BER and EVM results exhibited very good matching with the corresponding values from the graphs on Fig. 7, as it is presented particularly for the 4-QAM modulation case in Table I.

Table I. LTE downlink residual BER and average data EVM

BER	4. 1846E-3
Data_Avg_EVM %	8.9125

IV. Conclusion

The residual BER – error floor, though useful and widely used metrics for the end-to-end digital radio transmission performance metrics, provides no insight into the error-generating analog impairments that are easy to identify (and consequently be able to deal with) by the VSA tools such as polar I-Q vectors trajectories and constellation analysis. However, even this cannot help notice minor distortions that can still seriously degrade the performance, so the only option is the Error Vector Magnitude (EVM) analysis, which has therefore become very popular figure of merit in this regard.

So, we considered estimating EVM from the residual BER assuming that the latter's generally non-linear and non-additive "constituents" are substituted by the equivalent additive Gaussian noise source producing the same BER (and EVM) degradation. The resulting waterfall-shaped EVM(BER) curves were verified in the LTE lab to be a very good first approximation of EVM from the available residual BER value, when (expensive) VSA analysis tools are not available.

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